MA/CSSE 474 Day 28 Summary

1) **Turing machine (TM)** intro
   a) Tape alphabet, blank symbol, two-way-infinite tape, read/write head.
   b) **Transition:** based on current state and tape symbol, the TM
      i) Changes to next state
      ii) Writes a symbol on current tape square
      iii) Moves left or right (
         (1) In some other authors' equivalent TM models, staying on same square is option. Not here.
   c) **Formal TM definition.** A deterministic TM M is (K, Σ, Γ, δ, s, H):
      i) K is a finite set of states;
      ii) Σ is the input alphabet, which does not contain ☐;
      iii) Γ is the tape alphabet, which must contain ☐ and have Σ as a subset.
      iv) $s \in K$ is the initial state;
      v) $H \subseteq K$ is the set of halting states;
      vi) δ is the transition **function**: (for a nondeterministic TM, we will need a more general relation Δ)

$(K - H)$	$\times$	Γ	to	K	$\times$	Γ	$\times$	$\{\rightarrow, \leftarrow\}$
non-halting state	$\times$	tape char	$\rightarrow$	state	$\times$	tape char	$\times$	direction to move (R or L)

2) Example: M takes as input a string in the language: $\{a^i b^j, 0 \le j \le i\}$, and adds b's as required to make the number of b's equal the number of a's.

M = ({1, 2, 3, 4, 5, 6}, {a, b}, {a, b, ❑, $, #}, δ, 1, {6}), where δ =

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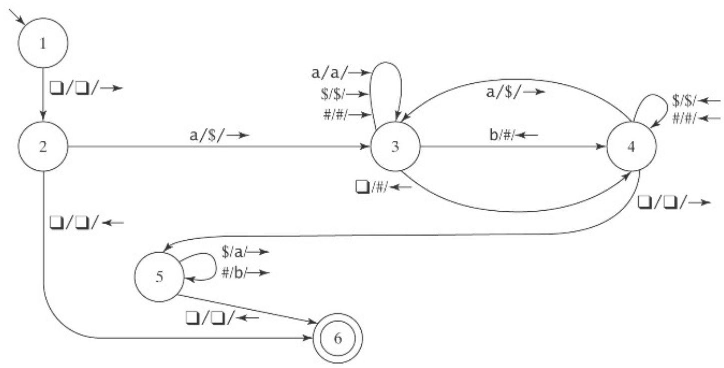
((1, ❑), (2, ❑, →)),
((1, a), (2, q, →)) — These four transitions are required because *M* must be defined for every state/ input pair, but since it isn't possible to see anything except ❑ in state 1, it doesn't matter what they do.
((1, b), (2, q, →)),
((1, $), (2, ❑, →)),
((1, #), (2, ❑, →)),
((2, ❑), (6, $, →)),
((2, a), (3, $, →)),
((2, b), (3, $, →)), — Three more unusable elements of δ. We'll omit the rest here for clarity.
((2, $), (3, $, →)),
((2, #), (3, $, →)),
((3, ❑), (4, #, ←)),
((3, a), (3, a, →)),
((3, b), (4, #, ←)),
((3, $), (3, $, →)),
((3, #), (3, #, →)),
((4, ❑), (5, ❑, →)),
((4, a), (3, $, →)),
((4, $), (4, $, ←)),
((4, #), (4, #, ←)),
((5, ❑), (6, ❑, ←)), — State 6 is a halting state and so has no transitions out of it
((5, $), (5, a, →)),
((5, #), (5, b, →)))



3) We will see how this machine works on several inputs.
4) **You should then trace its action on** aab:

5) Halting:
   a) DFSM:
   b) PDA:

   c) TM:

6) Formal definitions of TM computation. Most of this is like same definitions for FSM, PDA.
   a) Configuration: an element of

K ×	$((\Gamma - \{\square\})\, \Gamma^*) \cup \{\varepsilon\}$ ×	Γ ×	$(\Gamma^* (\Gamma - \{\square\})) \cup \{\varepsilon\}$
state	before current square	current square	after current square

   b) Abbreviation: (q, ab, b, b) can be abbreviated as (q, ab<u>b</u>b). (q, ε, □, abbb) as (q, <u>□</u>abbb)
   c) Initial configuration is always (*s*, <u>□</u>*w*). Tape head starts on the square before the input string.
   d) Yields (similar to FSM and PDA yields, but δ is different):
      i) $(q_1, w_1) \vdash_M (q_2, w_2)$ iff (q_2, w_2) is derivable, *via* δ, in one step.
      ii) For any TM *M*, let $\vdash_M$***** be the reflexive, transitive closure of $\vdash_M$.
      iii) Configuration C_1 ***yields*** configuration C_2 if: $C_1 \vdash_M^* C_2$.
      iv) A ***path*** through *M* is a sequence of configurations $C_0, C_1, \ldots, C_n$ for some $n \geq 0$ such that C_0 is the initial configuration and $C_0 \vdash_M C_1 \vdash_M C_2 \vdash_M \ldots \vdash_M C_n$.
      v) A **computation** by *M* is a path that halts.
      vi) If a computation is of *length n* (has *n* steps), we can write: $C_0 \vdash_M^n C_n$

Class exercise: Try to build this TM together: If *n* and *m* are non-negative integers, ***monus(n, m)*** *is defined to be* $n - m$ if $n > m$, and 0 if $n \leq m$.
Draw the diagram for a TM M whose input is $\mathbf{1^n;1^m}$ and whose output is $\mathbf{1^{monus(n,\,m)}}$.
When M halts, the read/write head should be positioned on the blank before the first 1.

For practice later: A TM to recognize { ww^R : w ∈ {a, b}* }.
If the input string is in the language, the machine should halt with y as its current tape symbol
If not, it should halt with n as its current tape symbol.
The final symbols on the rest of the tape may be anything.