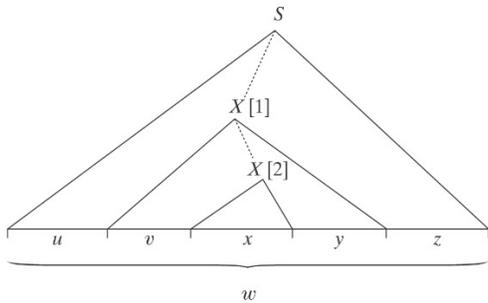**MA/CSSE 474   Day 26 Summary**

1) Context-free pumping theorem:

> If L is a context-free language, then
> $\exists k \geq 1$ ($\forall$ strings $w \in L$, where $|w| \geq k$
>   ($\exists u, v, x, y, z$ ($w = uvxyz$, $vy \neq \varepsilon$, $|vxy| \leq k$,
>   and
> $\forall q \geq 0$ ($uv^q xy^q z$ is in L)))).

2) As with the reg.-lang. pumping theorem, to show a language is *not* CF, we use the contrapositive. We do not get to choose the k or the breakdown into uvxyz. We choose the w∈L, and for each breakdown, q a such that $uv^q xy^q z \notin L$.

3) Make note of the slide on similarities and differences between the two pumping theorems.

4) $A^nB^nC^n = \{a^n b^n c^n, n \geq 0\}$ Three regions (1: all a's, 2: all b's, 3: all c's)

> **Contrapositive:** If
> $\forall k \geq 1$ ($\exists$ string $w \in L$, where $|w| \geq k$
>   ($\forall u, v, x, y, z$
>   ($w = uvxyz$,
> $vy \neq \varepsilon$,
> $|vxy| \leq k$, and
> $\exists q \geq 0$ ($uv^q xy^q z$ is not in L)))),
> then L is not a CFL.

5) $\{ a^{n^2} : n \geq 0 \}$

6) $L = \{a^n b^m a^n$, $n, m \geq 0$ and $n \geq m\}$.

Let $w = a^k b^k a^k$

          aaa … aaabbb … bbbaaa … aaa
            | 1 | 2 | 3 |

7) WcW = $\{wcw : w \in \{a, b\}^*\}$

8) $\{(ab)^n a^n b^n : n > 0\}$

9) $\{xcy : x, y \in \{0, 1\}^* \text{ and } x \neq y\}$

10) A PDA may never halt or never finish reading its input.

11) Nondeterminism can lead to exponential running time.

12) CFL closure:
   a) Union. New start symbol: add productions $S \rightarrow S_1$, $S \rightarrow S_2$

   b) Concatenation. New start symbol: add production $S \rightarrow S_1 S_2$

   c) Kleene Star. New start symbol: add productions $S \rightarrow \varepsilon$, $S \rightarrow S S_1$

   d) Reverse.   Transform grammar to Chomsky Normal form. Replace each production $A \rightarrow BC$ by $A \rightarrow CB$