MA/CSSE 474 Day 17 Summary

1. Another decision problem. On the slides; this space is for your notes on that solution.

2. CFG (context-free grammar intro)
   a. A ***rewrite system*** (a.k.a. ***production system*** or ***rule-based system***) is:
      i. a list of rules, and
      ii. an algorithm for applying them
   b. Simple-rewrite algorithm (given a string w and rewrite system R):
      i. Set working-string to w.
      ii. Until told by R to halt do:
         1. Match the lhs of some rule in R against some part of working-string.
         2. Replace the matched part of working-string with the rhs of the rule that was matched.
      iii. Return working-string.
   c. Example: w = SaS, Rules: $S \to aSb$, $aS \to \varepsilon$
      i. Questions the system must answer: Order to apply rules? When to quit?
   d. *Example:* $S \to aSb$, $S \to bSa$, and $S \to \varepsilon$
      i. Choices after: $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow$
   e. *Example:* $S \to aTTb$, $T \to bTa$, and $T \to \varepsilon$
      i. Choices after: $S \Rightarrow aTTb \Rightarrow$
   f. When to stop a derivation: (either of these)
      i. The working string no longer contains any nonterminal symbols (including, when it is ε).
      ii. There are nonterminal symbols in the working string but none of them is in a substring that is the left-hand side of any rule in the grammar.
      iii. Sometimes we can't stop!
   g. CFG G = (V, Σ, R, S), (each part is finite)
      i. Σ is the ***terminal alphabet***; it contains the symbols that make up the strings in *L*(*G*), and
      ii. *N is the* ***nonterminal alphabet*** a set of working symbols that G uses to structure the language. These symbols disappear by the time the grammar finishes its job and generates a string. **(Note:** $\Sigma \cap N = \varnothing$.) Rule alphabet: **V** = $\Sigma \cup N$
      iii. **R**: A set of rules (a.k.a. productions) of the form $A \to \beta$, where $A \in N$ and $\beta \in V^*$.
      iv. G has a unique **start symbol**, **S** ∈ N
   h. Let's write CFGs to generate:

A^nB^n **Bal** **$\{a^mb^n : m \geq n\}$**

3. In a regular grammar, every rule (production) in *R* must have a right-hand side that is
   a) ε, or
   b) a single terminal, or
   c) a single terminal followed by a single nonterminal.

$S \to bS$, $S \to aT$ $T \to aS$, $T \to b$, $T \to \varepsilon$

4. L is a regular language if and only if L = L(G) for some regular grammar G.