MA/CSSE 474 Day 14

1. Closure properties of regular languages: closed under union, concatenation, Kleene star, complement, difference, reverse, letter substitution (a function that maps each letter in Σ to a string in Σ').

2. In the homework you showed closure under intersection by construction. We can also do it using some other closure properties and DeMorgan's laws:

3. **Using closure backwards:** If $L_1 \cap L_2$ is regular, what can we say about L_1 and L_2?

   If L_2 is not regular, what can we say about L_1L_2?

4. **Showing that a language is *not* regular.**
   a. All non-regular languages are infinite.
   b. Where does "infiniteness" come from in a regular language?
      i. Regular expression approach:
      ii. DFSM approach:
   c. If a DFSM has k states and a string whose length is at least k …

   d. This is the basis for the "pumping theorem" (also known as the "pumping lemma") for regular languages.

5. Pumping theorem: **Informally**: If *L* is regular, then every long string in *L* is "infinitely pumpable (in and out)".

Formally, if L is regular, then
$\exists k \geq 1$ such that
(∀ strings $w \in L$,
($|w| \geq k$ →
($\exists\, x, y, z$ ($w = xyz$,
$|xy| \leq k$,
$y \neq \varepsilon$, and
$\forall q \geq 0$ (xy^qz is in L)))))

The contrapositive form:
(
)
)
)
)
)
→ L is not regular

6. Hopcroft and Ullman's "adversary argument" is a good way to understand this. The "adversary" is trying to show that L is regular; we are showing that it is not.
   a. Adversary picks k, x, y, z. We pick w, q. We must have a strategy for picking w and q that will work for any k and for any legal x, y, z.

7. Example: $\{a^nb^n : n \geq 0\}$ Details on slides. A place for notes:

8. Example: $\{a^n b^n : n \geq 0\}$ A different w. Details on slides. A place for notes:

9. Bal = $\{w \in \{\ ),\ (\ \}^* :$ the parens are balanced$\}$

10. PalEven = $\{ww^R : w \in \{a, b\}^*\}$

11. $\{a^n b^m : n \geq m\}$

12. $\{aba^n b^n : n \geq 0\}$