MA/CSSE 474 Day 09 Summary

1. **Myhill-Nerode Theorem:** A language is regular iff the number of equivalence classes of $\approx_L$ is finite.
    a. ***Proof:*** Show the two directions of the implication:
    b. ***L regular $\rightarrow$ the number of equivalence classes of $\approx_L$ is finite:*** If L is regular, then

    c. ***The number of equivalence classes of $\approx_L$ is finite $\rightarrow$ L regular:*** If the cardinality of $\approx_L$ is finite, then

The Algorithm *ndfsmtodfsm*

ndfsmtodfsm(M: NDFSM) =
1. For each state q in K_M do:
    1.1 Compute *eps*(q).
2. s' = *eps*(s)
3. Compute δ':
    3.1 *active-states* = {s'}.
    3.2 δ' = $\varnothing$.
    3.3 While there exists some element Q of *active-states* for which δ' has not yet been computed do:
        For each character c in Σ_M do:
            new-state = $\varnothing$.
            For each state q in Q do:
                For each state p such that $(q, c, p) \in \Delta$ do:
                    new-state = *new-state* $\cup$ *eps*(p).
            Add the transition (q, c, *new-state*) to δ'.
            If *new-state* $\notin$ *active-states* then insert it.
4. K' = *active-states*.
5. $A' = \{Q \in K' : Q \cap A \neq \varnothing\}$.

NDFSM->FSM algorithm proof.

2. **Statement:** From any NDFSM $M = (K, \Sigma, \Delta, s, A)$, *ndfsmtodfsm* constructs a DFSM $M' = (K', \Sigma, \delta', s', A')$, which is equivalent to M.
    a. $K' \subseteq \mathcal{P}(K)$ (power set of K)
    b. s' = *eps*(s)
    c. $A' = \{Q \subseteq K : Q \cap A \neq \varnothing\}$
    d. $\delta'(Q, a) = \cup \{eps(p): p \in K$ and $(q, a, p) \in \Delta$ for some $q \in Q\}$
3. **M' is deterministic.** δ' is defined for each reachable state of K' and for each alphabet symbol. And for each state q and symbol a, step 3.3 assigns only one value to $\delta'(q, a)$.
4. The hard part is showing that **L(M') = L(M)**.
5. **A useful Lemma:** (If this is all we get to today, it's okay, because this is good practice for the exam).
    a. Let w be any string in Σ^*, let p and q be any states in K, and let P be any state in K'. Then: $(q, w) \mid\!-_M^* (p, \varepsilon)$ **iff** $((eps(q), w) \mid\!-_{M'}^* (P, \varepsilon)$ where $p \in P)$.
    b. We will only need this lemma for the case q=s, but the more general statement is easier to prove by induction.
    c. **Informal restatement of the lemma:** If the original NDFSM M starts in state q and, after reading the string w, can land in state p (along at least one of its paths), then the new DFSM M' must behave as follows:

    The only-if part implies:

6. **The lemma:** $(q, w) \mid\!-_M^* (p, \varepsilon)$ **iff** $((eps(q), w) \mid\!-_{M'}^* (P, \varepsilon)$ and $p \in P)$ Proof: induction on |w|.
7. **Base case**: w = ε
    a. **if part:** $(eps(q), \varepsilon) \mid\!-_{M'}^* (P, \varepsilon) \wedge p \in P \quad \rightarrow \quad (q, \varepsilon) \mid\!-_M^* (p, \varepsilon)$

b. **only if part:** $(q, \varepsilon) \vdash_{M}^{*} (p, \varepsilon) \rightarrow (eps(q), \varepsilon) \vdash_{M'}^{*} (P, \varepsilon)$ and $p \in P$

8. **Induction step:** Let w have length $k + 1$. Then w = zc where $z \in \Sigma^*$ has length k, and $c \in \Sigma$.
    a. **Induction assumption.** The lemma is true for z.
    b. **To show:** The lemma is true for w.

I.e. Assume that M and M' "behave identically" for the first k input characters: show for k+1

9. **This boils down to showing:**
   $(q, zc) \vdash_{M}^{*} (s_i, c) \vdash_{M}^{*} (p, \varepsilon)$ iff $(eps(q), zc) \vdash_{M'}^{*} (Q, x) \vdash_{M'} (P, \varepsilon)$ and $p \in P$
    a. **If part:** $(eps(q), zc) \vdash_{M'}^{*} (Q, c) \vdash_{M'} (P, \varepsilon)$ and $p \in P \rightarrow (q, zc) \vdash_{M}^{*} (s_i, c) \vdash_{M}^{*} (p, \varepsilon)$

    b. **Only if part:** $(q, zc) \vdash_{M}^{*} (s_i, c) \vdash_{M}^{*} (p, \varepsilon) \rightarrow (eps(q), zc) \vdash_{M'}^{*} (Q, c) \vdash_{M'} (P, \varepsilon)$ and $p \in P$

10. **Using the lemma to prove the theorem** is straightforward.
    Recall the theorem: L(M')=L(M). There are two things to show (details are on the last two slides, in case there is not time to write them down before class ends)
    a. L(M) ⊆ L(M'). Suppose that $w \in L(M)$ (original machine M accepts w):

    b. L(M') ⊆ L(M). Prove the contrapositive. Suppose that $w \notin L(M)$ (i.e. the original NDFSM does not accept w):