MA/CSSE 474 Day 03 Summary . Main ideas from today (and probably a few that we'll get to tomorrow):

1. Operations on strings:
   a. |w|
   b. $\#_a(w)$
   c. Concatenation wx (it's associative, and ε is the identity for this operation)
   d. w^i
   e. w^R (recursive definition): $\varepsilon^R = \varepsilon$, $(ua)^R = au^R$

2. ***Theorem:*** If w and x are strings, then $(wx)^R = x^R w^R$. Prove it by induction on |x|

   Base case: $|x| = 0$: Then $x = \varepsilon$, and $(wx)^R = (w\,\varepsilon)^R = (w)^R = \varepsilon\, w^R = \varepsilon^R\, w^R = x^R\, w^R$.

   Induction step: $\forall n \geq 0\ (((|u| = n) \rightarrow ((w\,u)^R = u^R\, w^R)) \rightarrow ((|x| = n + 1) \rightarrow ((w\,x)^R = x^R\, w^R)))$:

   Consider any string x, where $|x| = n + 1$. Then $x = u\,a$ for some symbol a and $|u| = n$. So:

$(w\,x)^R$	$= (w\,(u\,a))^R$	rewrite x as ua
	$= ((w\,u)\,a)^R$	associativity of concatenation
	$= a\,(w\,u)^R$	definition of reversal
	$= a\,(u^R\,w^R)$	induction hypothesis
	$= (a\,u^R)\,w^R$	associativity of concatenation
	$= (ua)^R\,w^R$	definition of reversal
	$= x^R\,w^R$	rewrite ua as x

3. You should know the meaning of *substring*, *prefix*, and *suffix*, and the "proper" version of each of those.

4. Describe in simple English the language $L = \{x : \exists u \in \{a, b\}^* : x = ua\}$

5. $L = \{x\#y$: $x, y \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}^*$ and, when x and y are viewed as the decimal representations of natural numbers, *square*(x) = $y\}$.
   Examples: (in L or not?) : #9, 12#144, 3#8, 12, 12#12#12, #

6. Give a very short symbolic description of $L = \{a^n : n \geq 0\}$

7. A language that we will use a lot: $A^nB^n = \{a^kb^k : k \geq 0\}$

8. Is the Halting problem language well-specified? Can we decide which strings it contains?

   L = {w: w is a Java program that, no matter what finite input string it is given, is guaranteed to halt}.

9. Languages based on prefixes :
   $L = \{w \in \{a, b\}^*$: no prefix of w contains b$\}$

   $L = \{w \in \{a, b\}^*$: no prefix of w starts with a$\}$

   $L = \{w \in \{a, b\}^*$: every prefix of w starts with a$\}$

10. Concatenation of languages, and related things
    a. L_1L_2. If L1 = {a, aa}, and L2 = {a, c, ε}, then L1 L2 = . What is L{ε}? LØ?
    b. L^R
    c. L^3 Is this the same as {w3 : w ∈ L } ?
    d. L^0
    e. L^k
    f. L*
    g. L^+

11. How many different partitions of the set {1, 2, 3}?

12. Are the natural numbers closed under f(x) = 1+x? g(x) = x – 1? h(x) = 1/x k(x) = # distinct factors of x?

13. What is the closure of $\mathbb{N}^+$ under division?

14. Show that $\equiv_3$ is an equivalence relation. (a $\equiv_3$ b iff b-a = 3k for some integer k).

    reflexive:

    symmetric:

    transitive:

15. Can a language be uncountable?

    Is the set of languages over a specific alphabet uncountable?

16. Questions about the *maxstring* function (defined on a slide):
    maxstring(A^nB^n) = *maxstring*({a}*) =
    Let INF be the set of infinite languages. Let FIN be the set of finite languages.
    Are the language classes FIN and INF closed under *maxstring*?

17. Questions about the *chop* function (defined on a slide):
    What is *chop*(A^nB^n)? What is *chop*($A^nB^nC^n$)?
    Are FIN and INF closed under *chop*?