474 HW 3 problems (highlighted problems are the ones to turn in)

If you need simpler practice proiblems (and you probably do!), do some other parts of 5.2 first.

5.2

2. Show a DFSM to accept each of the following languages:
   a. $\{w \in \{a, b\}^* :$ every a in w is immediately preceded and followed by b$\}$.
   b. $\{w \in \{a, b\}^* : w$ does not end in ba$\}$.
   c. $\{w \in \{0, 1\}^* : w$ corresponds to the binary encoding, without leading 0's, of natural numbers that are evenly divisible by 4$\}$.
   d. $\{w \in \{0, 1\}^* : w$ corresponds to the binary encoding, without leading 0's, of natural numbers that are powers of 4$\}$.
   e. $\{w \in \{0\text{-}9\}^* : w$ corresponds to the decimal encoding, without leading 0's, of an odd natural number$\}$.
   f. $\{w \in \{0, 1\}^* : w$ has 001 as a substring$\}$.
   g. $\{w \in \{0, 1\}^* : w$ does not have 001 as a substring$\}$.
   h. $\{w \in \{a, b\}^* : w$ has bbab as a substring$\}$.
   i. $\{w \in \{a, b\}^* : w$ has neither ab nor bb as a substring$\}$.

5.2j

   j. $\{w \in \{a, b\}^* : w$ has both aa and bb as substrings$\}$.
   k. $\{w \in \{a, b\}^* : w$ contains at least two b's that are not immediately followed by an a$\}$.

5.2l

   l. $\{w \in \{0, 1\}^* : w$ has no more than one pair of consecutive 0's and no more than one pair of consecutive 1's$\}$.
   m. $\{w \in \{0, 1\}^* :$ none of the prefixes of w ends in 0$\}$.
   n. $\{w \in \{a, b\}^* : (\#_a(w) + 2 \cdot \#_b(w)) \equiv_5 0\}$. ($\#_a(w)$ is the number of a's in w).

5.3

3. Consider the children's game Rock, Paper, Scissors. We'll say that the first player to win two rounds wins the game. Call the two players A and B.
   a. Define an alphabet Σ and describe a technique for encoding Rock, Paper, Scissors games as strings over Σ. (*Hint:* Each symbol in Σ should correspond to an ordered pair that describes the simultaneous actions of A and B.)
   b. Let L_{RPS} be the language of Rock, Paper, Scissors games, encoded as strings as described in part (a), that correspond to wins for player A. Show a DFSM that accepts L_{RPS}.

5.4

4. If M is a DFSM and $\varepsilon \in L(M)$, what simple property must be true of M?

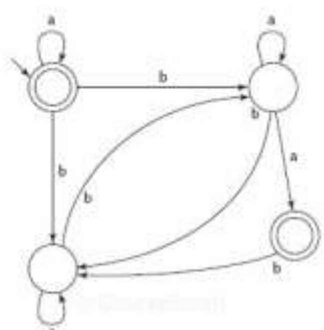
The answer is simple and straightforward, so don't look for anything complicated or tricky.

Problem 5 (On the assignment sheet, not from textbook. DFSM for "divisible by 3")

5. (t-6)Let L be $\{w \in \{0, 1\}^* : \exists n, k \in \mathbb{N}\ (w = \langle n \rangle \wedge n = 3k)\}$. I.e. the set of binary representations of natural numbers that are divisible by 3. Leading zeroes are allowed. Recall that $0 \in \mathbb{N}$. Draw the transition diagram or a transition table for a DFSM that accepts L. [Hint: think about remainders *mod* 3. Another hint: There are not many states].

5.5

5. Consider the following NDFSM M:



For each of the following strings w, determine whether $w \in L(M)$:
   a. aabbba.
   b. bab.
   c. baba.

5.6a

5.6c

5.7

6. Show a possibly nondeterministic FSM to accept each of the following languages:

a. $\{a^n b a^m : n, m \geq 0, n \equiv_3 m\}$.

b. $\{w \in \{a, b\}^* : w$ contains at least one instance of aaba, bbb or ababa$\}$.

c. $\{w \in \{0\text{-}9\}^* : w$ corresponds to the decimal encoding of a natural number whose encoding contains, as a substring, the encoding of a natural number that is divisible by 3$\}$.

d. $\{w \in \{0, 1\}^* : w$ contains both 101 and 010 as substrings$\}$.

e. $\{w \in \{0, 1\}^* : w$ corresponds to the binary encoding of a positive integer that is divisible by 16 or is odd$\}$.

f. $\{w \in \{a, b, c, d, e\}^* : |w| \geq 2$ and w begins and ends with the same symbol$\}$.

7. Show an FSM (deterministic or nondeterministic) that accepts $L = \{w \in \{a, b, c\}^* : w$ contains at least one substring that consists of three identical symbols in a row$\}$. For example:

- The following strings are in L: aabbb, baacccbbb.
- The following strings are not in L: ε, aba, abababab, abcbcab.

5.6(c)
Note that this one is decimal, while problem 1 is binary. Also notice the "contains a substring" part.