4.4

Are the following sets closed under the following operations? Prove your answer. If a set is not closed under the operation, what is its closure under the operation? I suggest that students look over all of these answers, to firm up the notion of a set of languages being closed under an operation.

a) $L = \{w \in \{a, b\}^* : w \text{ ends in } a\}$ under the function *odds*, defined on strings as follows: *odds*(*s*) = the string that is formed by concatenating together all of the odd numbered characters of *s*. (Start numbering the characters at 1.) For example, *odds*(ababbbb) = aabb.

Not closed. If $|w|$ is even, then the last character of *odds*(*w*) will be the next to the last character of *w*, which can be either a or b. For any *w*, $|odds(w)| \leq |w|$, and the shortest string in *L* has length 1. So the closure is $\{a, b\}^+$.

b) FIN (the set of finite languages) under the function *oddsL*, defined on languages as follows:

$oddsL(L) = \{w : \exists x \in L\ (w = odds(x))\}$

FIN is closed under the function *OddsL*. Each string in *L* can contribute at most one string to *OddsL*(*L*). So $|OddsL(L)| \leq |L|$.

d) FIN under the function *maxstring*, defined in Example 8.22.

FIN is closed under the function *maxstring*. Each string in *L* can contribute at most one string to *maxstring*(*L*). So $|maxstring(L)| \leq |L|$.

e) INF under the function *maxstring*.

INF is not closed under *maxstring*. a* is infinite, but $maxstring(a^*) = \emptyset$, which is finite.