

## 474 Instructor Notes from Day 17 slides:

Slide 7: Using the Closure Properties

Answer:  $a^*b^*$

Slide 10:  $L = \{a^i b^j : i, j \geq 0 \text{ and } i \neq j\}$

If  $\neg L$  were regular, then the intersection of  $\neg L$  and  $a^*b^*$  would be regular. In a previous slide, we showed that it is not regular.

Slide 21: Defining Functions from one Language to Another

Let  $w$  be any string from  $a^*db^*$ .  $|w|$  is even, it contributes nothing to  $\text{chop}(L)$ .

If  $|w|$  odd, and  $\#a$ 's =  $\#b$ 's. then  $w$  contributes  $a^n b^n$  to  $\text{chop}(L)$ , for some  $n$ .

otherwise,  $|w|$  is odd, and  $d$  is not in the middle, so  $\text{chop}$  removes an  $a$  or  $b$ .

Since  $|w|$  is odd difference between  $\#a$  and  $\#b$  must be at least 2, so chopped string also has different number of  $a$ 's and  $b$ 's.

$\text{Chop}(a^*db^*)$  contains all strings from  $A^n B^n$ , plus some strings in  $\{a^*bb^*$  whose length is even}.

Can it be regular? If so, its intersection with  $a^*b^*$  would be regular. But that intersection is  $A^n B^n$

Slide 29: Totality

Construct  $M'$  to accept  $\neg L(M)$ .

2. Return  $\text{emptyFSM}(M')$ .

Slide 30: Finiteness:

The mere presence of a loop does not guarantee that  $L(M)$  is infinite. The loop might be:

- labeled only with  $\epsilon$ ,
- unreachable from the start state, or
- not on a path to an accepting state.

1.  $M' = \text{ndfsmtodfsm}(M)$ .

2.  $M'' = \text{minDFSM}(M')$ .

3. Mark all states in  $M''$  that are on a path to an accepting state.

4. Considering only marked states, determine whether there are any cycles in  $M''$ .

5. If there are cycles, return *True*. Else return *False*.

The simulation approach:

$M' = \text{ndfsmtodfsm}(M)$ .

2. For each string  $w$  in  $\Sigma^*$  such that \_\_\_\_\_ do: [answer:  $|K_{M'}| \leq w \leq 2 \cdot |K_{M'}| - 1$  ]  
Run  $\text{decideFSM}(M', w)$ .

3. If  $M'$  accepts at least one such string, return *False*.  
Else return *True*.

Slide 34: Minimality

$M' = \text{minDFSM}(M)$ .

2. If  $|K_M| = |K_{M'}|$  return *True*; else return *False*.