Slide 3: How many languages are there?
IThe answer is no, so $L_{d}$ is not in the enumeration, and the set of languages is uncountable.

Write "Floyd-Warshall" on the board.

## Slide 4: How Many Regular Languages?

How do we know that there are countably many regular expressions over a given alphabet?
We can easily enumerate them, based on the number of rules involved in the construction.

## Slide 8: Regular Does Not Always Mean Tractable

How many states are there? Each disk is on one of the three poles. The order of the disks on a given pole is fixed. So there are $3^{64}$ states.

## Slide 10: Closure Properties of Regular Languages

## Reverse:

By construction. Let $M=(K, \Sigma, \delta, s, A)$ be any FSM that accepts $L$. $M$ must be written out completely, without an implied dead state. Then construct $M^{\prime}=\left(K^{\prime}, \Sigma^{\prime}, \delta^{\prime}, s^{\prime}, A^{\prime}\right)$ to accept reverse $(L)$ from $M$ :

Initially, let $M^{\prime}$ be $M$.
Reverse the direction of every transition in $M^{\prime}$.
Construct a new state $q$. Make it the start state of $M^{\prime}$. Create an $\varepsilon$-transition from $q$ to every state that was an accepting state in $M$.
$M^{\prime}$ has a single accepting state, the start state of $M$.
Slide 11: Closure of Regular Languages under intersection
$L(M 1) \cap L(M 2)=\neg(\neg L(M 1) \cup \neg L(M 2))$.
In the homework, you will directly construct a machine to do the intersection
Slide 12: Closure of Regular Languages Under Difference

$$
\mathrm{L}(\mathrm{M} 1)-\mathrm{L}(\mathrm{M} 2)=\mathrm{L}(\mathrm{M} 1) \cap \neg \mathrm{L}(\mathrm{M} 2) .
$$

## Slide 16: Don't try to use closure backwards

Also intersection of $\left\{a^{n} b^{n}\right\}$ and $\left\{a^{n} b^{n+1}\right\}$ is empty, hence regular.

