MA/CSSE 474 Exam 3 Winter 2009-10 Name_____

Closed book and notes, except for three 8.5 x 11 sheets of paper (can be 2-sided).

No electronic devices, especially ones with headphones.

Scores:

Problem	Possible	Score
1	28	
2	32	
3	10	
4	10	
5	15	
6	10	
7	10	
8	5	
Total	120	

1. (28 points) For each of the following statements, circle T or F to indicate whether it is *True* or *False*.
If it is sometimes False, you should choose False. You do not have to give proofs or counterexamples.
For each part, you get 2 points for circling IDK (I don't know), 4 for circling the correct answer, and 0 for circling the incorrect answer or leaving it blank. Reason: When you don't know something, knowing that you don't know counts for something.

   a) T F IDK If R is regular and R∩L is context-free, then L is context-free
   b) T F IDK If R is regular and R∩L is not context-free, then L is not context-free
   c) T F IDK The complement of a context-free language cannot be context-free.
   d) T F IDK Every context-free language is decidable.
   e) T F IDK Let L be such that, for each $w \in L$, there exists some DFSM that accepts w. Then L must be regular.
   f) T F IDK If L^+ is context-free, then L must be context-free.
   g) T F IDK If L is context-free, then L^+ must be context-free.

2. (32 points) For each of the following statements, circle
R if the language is regular,
CF-R if it is context-free but not regular,
NCF if it is not context-free
IDK if you don't know. **Scoring:** Correct answer - 4, IDK - 2, incorrect answer - 0.

   a) R CF-R NCF IDK $WW^R = \{ww^R : w \in \{a,b\}^*\}$.
   b) R CF-R NCF IDK {u#v^R : u and v are binary encodings (no leading zeroes) of positive integers, where v=2u}
   c) R CF-R NCF IDK {u#v : u, v ∈ {a, b}* and ∃x∈{a, b}* (x is a substring of u and x^R is a substring of v)
   d) R CF-R NCF IDK $\{w = xyz : x \in 0^*, y \in 1^*, z \in 0^*, |x| = |z| \text{ and } |y| = 2 \cdot |z|\}$
   e) R CF-R NCF IDK L(G) where G is S→TSb |Tb, T→Ta | ε .
   f) R CF-R NCF IDK $\neg L$, where $L = \{w \text{c} w^R : w \in \{\text{a}, \text{b}\}^*\}$.
   g) R CF-R NCF IDK $\{\text{a}^i\text{b}^j\text{c}^k : i, j, k \geq 0 \text{ and } 2i + 3j \equiv_3 k\}$ [recall that $\equiv_3$ means "congruent mod 3"]
   h) R CF-R NCF IDK {$w \in \{\text{a}, \text{b}, \text{c}\}^*$: every a has a matching b and a matching c somewhere in w, and no b or c is considered to match more than one a}

3. (10 points) Choose a language from problem 2 that is not context-free, and prove that it is not CF. You must use the Pumping Theorem in your proof. [**Hint** (if you want it): For 3 points (plus I will immediately grade that part of problem 2), I will tell you which part of #2 I think is the easiest one to use for this problem. And you will know that the language in that problem is not CF.[

☐ Student "bought" the hint. Which part are you doing? _____

4. (10 points) Design a Turing Machine that computes *n* % *m* (i.e., the remainder when integer *n* is divided by integer *m*) in unary. If the input is $\mathbf{1^n,1^m}$ (the comma is part of the input string), the output should be $\mathbf{1^{n\%m}}$. Your description may include a transition diagram, one or more of our macro diagrams, and/or a clear English description of how your machine works. The burden is on you to convince me that your machine works.

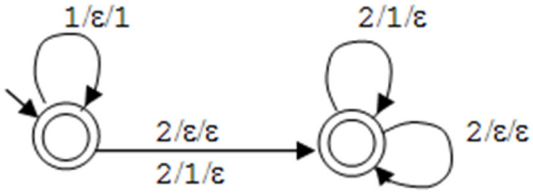
5. (15 points) Let $L = \{a^m b^{2n} c^{3n} d^p : p > m$, and $m, n \geq 1\}$

   a) (3) What is the shortest string in L?

   b) (6) Write a context-free grammar that generates L.

   c) (6) Define a pushdown automaton that accepts L.

6. (10 points) Consider the following PDA:



   a) (3) Give a concise description of $L(M)$.

   b) (3) Is M deterministic? Justify your answer.

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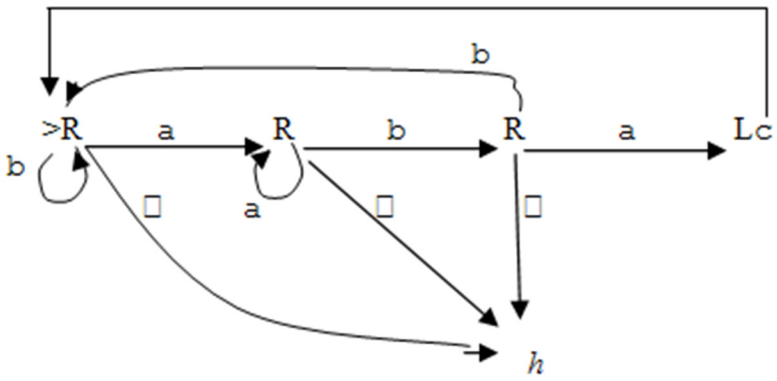
   c) (4) Is $L(M)$ deterministic context-free? Justify your answer.

7. (10 points) Give a short but careful English description of what this TM does.



8. (5 points) Where does the "k" in the Pumping Theorem for context-free languages come from?
[Hint: for a regular language, k is the number of states in a DFSM that recognizes the language.]