MA/CSSE 474 Exam 2 Jan 21, 2014 Name______________________ Section: 02 (3^{rd})___ 03(4^{th}) ___

Closed book and notes, except for two 8.5 x 11 sheets of paper (can be 2-sided).

Put your name on that paper and turn it in (in a separate pile) when you turn in your exam paper.

No electronic devices; in particular you may not use anything with headphones or earbuds.

Use your time wisely. First do the problems that you are sure you can do quickly and correctly.

Problem	Possible	Score
1	60	
2a	15	
2b	15	
3	25	
4a	10	
4b	15	
5	15	
6	15	
Total	**170**	

1. (60 points) Circle T or F to indicate whether it is *True* or *False*. If you circle IDK, it means "I don't know" If the statement is sometimes False, then False is the correct answer. **You need not give proofs or counterexamples here.**
For each part, you earn **3 points** for circling IDK, **6 points** for circling the correct answer, **-2 points** for circling the incorrect answer, and **0 points** if you leave it blank. Leaving it blank is silly, since you get more points for IDK.

a) T F IDK If the alphabet Σ contains at least two symbols, there are infinitely many uncountable languages over that alphabet.

b) T F IDK If the alphabet Σ contains at least two symbols, the number of regular languages over that alphabet is uncountable.

c) T F IDK The complement of a non-regular language L cannot be regular.

d) T F IDK Every finite language is regular.

e) T F IDK The union of a countable number of regular languages must be regular.

f) T F IDK The language $\{xy : x, y \in \{a, b\}^*$ and $|x| = |y|\}$ is regular.

g) T F IDK The language $\{xyx^R : x \in \{0, 1\}^+$ and $y \in \{0, 1\}^*\}$ is regular

h) T F IDK The language $\{xy : x, y \in \{a, b\}^*$ and $\exists z$ (z is a prefix of y and $z = as$ for some $s \in \{a, b\}^*)\}$ is regular.

i) T F IDK The language $\{w \in \{a, b\}^* :$ for each suffix x of w, $\#_a(x) \geq \#_b(x)\}$ is regular.

j) T F IDK The set of non-regular languages is closed under intersection.

2. L = $\{ w \in \{a, b\}^* : (|w|$ is even$) \rightarrow (w$ contains an even number of a's$) \}$

(15) Show that L is regular by using its definition and some closure properties of regular languages.

(15) Show that L is regular directly (***via*** a Finite State Machine or a regular expression)

3. (25) Use the pumping theorem to show that the following language is *not* regular. Include enough words to convince me that you understand the logic of using the theorem.

$L = \{a^i b^n : i, n > 0 \text{ and } (i = n \text{ or } i = 2n)\}$

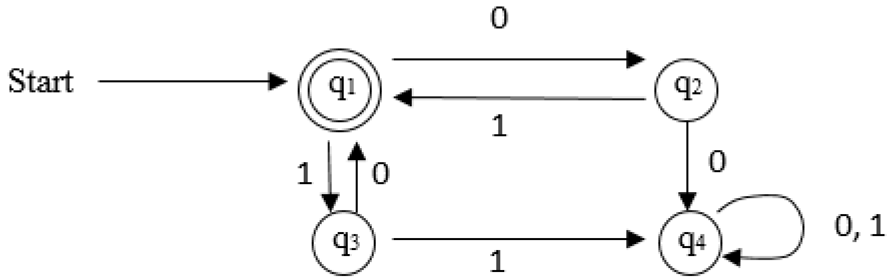
4. We can define *mid* to be a function on **strings**: For any string s over alphabet Σ:
   - If $|s| \leq 2$ then $mid(s) = \varepsilon$.
   - If $|s| > 2$ then let x and z be the single symbols from Σ that begin and end s. I.e. $s = xwz$ for some $w \in \Sigma^*$. Then $mid(s) = w$.

   For any **language** L over Σ, we define the function $midL(L)$ as follows:
   $midL(L) = \{t \in \Sigma^* : t = mid(s) \text{ for some } s \in L\}$

(10) If L is the language that is denoted by the regular expression **a*ba*b**,
write a regular expression that denotes the language $midL$(L).

(15) Show that the set of non-regular languages is *not* closed under *midL*.
Hint: Consider $L = \{ba^pb: p > 0 \text{ and p prime}\} \cup \{ca^pc: p > 0 \text{ and p not prime}\}$.

5. (15) In HW6, you found a regular expression that denotes the language accepted by the following DFSM:



Here are my calculations of some of the r_{ijk} values from my HW 6 solutions:
You may assume that the table is correct.

The question:
What is r_{223}?
If you correctly use the formula from class, you do not have to simplify the regular expression that you get.

	k=0	k=1	k=2
r_{11k}	ε	ε	(01)*
r_{12k}	0	0	0(10)*
r_{13k}	1	1	(01)*1
r_{14k}	∅	∅	0(10)*0
r_{21k}	1	1	(10)*1
r_{22k}	ε	ε ∪ 10	(10)*
r_{23k}	∅	11	(10)*11
r_{24k}	0	0	0
r_{31k}	0	0	0(01)*
r_{32k}	∅	00	00(10)*
r_{33k}	ε	ε∪01	ε∪0(01)*1
r_{34k}	1	1	1
r_{41k}	∅	∅	∅
r_{42k}	∅	∅	∅
r_{43k}	∅	∅	∅
r_{44k}	ε ∪ 0 ∪ 1	ε ∪ 0 ∪ 1	ε ∪ 0 ∪ 1

6. (15) Define a decision procedure to answer the following question for DFSMs over Σ = {a, b}:
Given a DFSM *M*, does *L*(*M*) contain at least one string that begins with *ab*?

Your procedure may (and probably should) use (i.e. call) algorithms from the textbook and the homework.