MA/CSSE 474 Exam 2 Winter 2009-10 Name________________________

Closed book and notes, except for three 8.5 x 11 sheets of paper (can be 2-sided).

No electronic devices, especially ones with headphones.

Scores:

Problem	Possible	Score
1	48	
2	15	
3	10	
4	10	
5a	10	
5b	10	
5c	10	
6	7	
Total	120	

1. (48 points) For each of the following statements, circle T or F to indicate whether it is *True* or *False*.
If it is sometimes False, you should choose False.
You do not have to give proofs or counterexamples.
For each part, you get 1 point for circling IDK (I don't know), 3 for circling the correct answer, and 0 for circling the incorrect answer. Reason: When you don't know something, knowing that you don't know counts for something.

a) T F IDK $(a \cup b)\, a^* = (\varepsilon \cup b)\, a^*$ (equal in the sense that they define the same language).
b) T F IDK $L(\varepsilon^*) \cap L(\varnothing^*) = \varnothing$.
c) T F IDK $\{a^n b^* a^n : n \geq 0\} \cap \{b^n a^* b^n : n \geq 0\}$ is a regular language.
d) T F IDK $\{w \in \{a, b, c\}^* : (|w| \text{ is even}) \rightarrow (w$ contains an even number of a's$\}$ is a regular language.
e) T F IDK $\{w = xy, x \in a^*b, y \in a^*b, |x| = |y|\}$ is a regular language.
f) T F IDK $\{(abc)^n a^n : n \geq 0\}$ is a regular language.
g) T F IDK $\{x\#y: x, y \in \{a, b\}^*, |x| + |y| \geq 4\}$ is a regular language.
h) T F IDK Let $L = \{a^p : p$ is a prime integer$\}$. L* is a regular language.
i) T F IDK If L is regular and L ∩ M is regular, then M must be regular.
j) T F IDK If L is regular and L ∩ M is not regular, then M cannot be regular.
k) T F IDK If L and M are regular, then N = $\{x : x \in L \text{ and } x^R \in M\}$ must be regular.
l) T F IDK The number of regular languages over the alphabet $\Sigma = \{a, b\}$ is countable.
m) T F IDK Let M be a DFSM such that $|K_M| = 100$, $\Sigma_M = \{a, b\}$, and $L(M)$ is finite.
We cannot tell whether $w = a^{100}b^{100} \in M$ without running M on w.
n) T F IDK The nonregular languages are closed under complement.
o) T F IDK Let L be such that, for each $w \in L$, there exists some DFSM that accepts w. Then L must be regular.
p) T F IDK There is an infinite number of uncountable languages.

2. (15 points) Let *mid* be a function on strings, defined as follows:
For any string *s* in some language *L* over Σ:

- If $|s| \leq 2$ then $mid(s) = \varepsilon$.
- If $|s| > 2$ then let *x* and *z* be single characters in Σ.
  Then we can rewrite *s* as *xwz* for some $w \in \Sigma^*$. $mid(s) = w$.

For any language *L* over Σ, we can define the function *midL*(*L*) as follows:

$$midL(L) = \{t \in \Sigma^* : t = mid(s) \text{ for some } s \in L\}$$

a) (2) What is *midL*(a*ba*)?

b) (3) Is FIN (the set of finite languages) closed under *midL*? YES NO
Explain briefly.

c) (5) Is INF (the set of infinite languages) closed under *midL*? YES NO
Explain briefly.

d) (5) Is the set of regular languages closed under *midL*? YES NO
Explain your answer.

3. (10 points) Give a decision procedure for the following problem:
Let $\Sigma = \{a, b\}$. Given an FSM M, does $L(M)$ contain at least one string that starts with *ab*?

4. (10 points) Show a context-free grammar that generates $\{a^n b^m : m \geq n \text{ and } m-n \text{ is odd}\}$:

5. (30 points) For each part of this problem, choose one of the following parts of problem 1 (circle its letter here).
Then circle the answer that you gave for that part.
Finally, give a proof of the answer that you gave. Your proof does not have to be formal, but it must be convincing.
You will not get credit for all three parts of this problem if you do not choose at least one part for which you answered T and at least one part for which you answered F.
No credit for a part of this problem if your answer for the corresponding part in #1 is incorrect.
I may give a few extra-credit points for any part if you choose one of the harder ones and explain it very well.

a) Part: c d e f g h Answer: T F
Proof:

b) Part: c d e f g h Answer: T F
Proof:

c) Part: c d e f g h Answer: T F
Proof:

6. (7 points) The table below shows the DFSM and my partial solution to a problem from HW7 (calculating a regular expression that defines the language that is recognized by a given DFSM). Your job is to use these results to compute the value of r_{223}. Be sure that your answer demonstrates that you correctly use the recursive formula.
You are not required to simplify your answer.

r_{223} =

	k=0	**k=1**	**k=2**
r_{11k}	ε	ε	$\varepsilon \cup 0(10)^*1 = (01)^*$
r_{12k}	0	0	$0 \cup 0(\varepsilon \cup 10)^* (\varepsilon \cup 10) = 0(10)^*$
r_{13k}	1	1	$1 \cup 0(\varepsilon \cup 10)^*11 = 1 \cup (01)^+1 = (01)^*1$
r_{14k}	$\varnothing$	$\varnothing$	$0(10)^*0$ Here and in later rows, I skip the $(\varepsilon \cup 10)^* = (10)^*$ step
r_{21k}	1	1	$(10)^*1$
r_{22k}	ε	$\varepsilon \cup 10$	$(10)^*$
r_{23k}	$\varnothing$	11	$11 \cup (\varepsilon \cup 10)(10)^*11 = (10)^*11$
r_{24k}	0	0	0
r_{31k}	0	0	$0 \cup 00(10)^*1 = 0(\varepsilon \cup 0(10)^*1) = 0(01)^*$
r_{32k}	$\varnothing$	00	$00 \cup 00(10)^* (\varepsilon \cup 10) = 00(10)^*$
r_{33k}	ε	$\varepsilon \cup 01$	$\varepsilon \cup 01 \cup 00(01)^*11 = \varepsilon \cup 0(\varepsilon \cup 0(01)^*1)1 = \varepsilon \cup 0(01)^*1$
r_{34k}	1	1	1
r_{41k}	$\varnothing$	$\varnothing$	$\varnothing$
r_{42k}	$\varnothing$	$\varnothing$	$\varnothing$
r_{43k}	$\varnothing$	$\varnothing$	$\varnothing$
r_{44k}	$\varepsilon \cup 0 \cup 1$	$\varepsilon \cup 0 \cup 1$	$\varepsilon \cup 0 \cup 1$