MA/CSSE 474 Exam 2 solutions Winter 2013-14

Problem 1 answers and reasons

a) T **F** IDK If the alphabet Σ contains at least two symbols, there are infinitely many uncountable languages over that alphabet. Every language over Σ is a set of finite strings over a finite alphabet. Such a set can be put into lexicographic order, so it is countable.

b) T **F** IDK If the alphabet Σ contains at least two symbols, the number of regular languages over that alphabet is uncountable. Each regular language over Σ is LM) for some canonical minimal DFSM M. Since each DFSM can be described by a (finite) string over Σ and a fixed set of other symbols, there are countably many DFSMs over Σ.

c) **T** F IDK The complement of a non-regular language L is never regular. Let L' be the complement of non-regular language L. Suppose L' is regular are closed under complement, the complement of :L' must be regular. But that complement Is L, which is not regular.

d) **T** F IDK Every finite language is regular. A regular expression for it is simply the union of all of the strings in the language.

e) T **F** IDK The union of a countable number of regular languages must be regular. Every single-element language is regular, and every language (regular or non-regular) is the countable union of its individual elements.

f) **T** F IDK The language $\{xy : x, y \in \{a, b\}^* \text{ and } |x| = |y|\}$ is regular. This is simply the language of even-length strings.

g) **T** F IDK The language $\{xyx^R : x \in \{0, 1\}^+ \text{ and } y \in \{0, 1\}^*\}$ is regular. This is simply the language strings whose first and last symbols match.

h) **T** F IDK The language $\{xy : x, y \in \{a, b\}^* \text{ and } \exists z\ (z \text{ is a prefix of } y \text{ and } z = as \text{ for some } s \in \{a, b\}^*)\}$ is regular. This is the language of all strings that contain at least one a.

i) T **F** IDK The language $\{w \in \{a, b\}^* : \text{for each suffix } x \text{ of } w, \#_a(x) \geq \#_b(x)\}$ is regular.

Not regular. We use the Pumping Theorem to show this. Let $w = b^k a^k$. $y = b^p$, for some nonzero p. Let q be 2. The resulting string is $b^{k+p}a^k$. It is a suffix of itself and the number of b's is greater than the number of a's. So it is not in L.

j) T **F** IDK The non-regular languages are closed under intersection. Consider the non-regular languages A^nB^n and C^nD^n. Their intersection is ∅, which is regular.

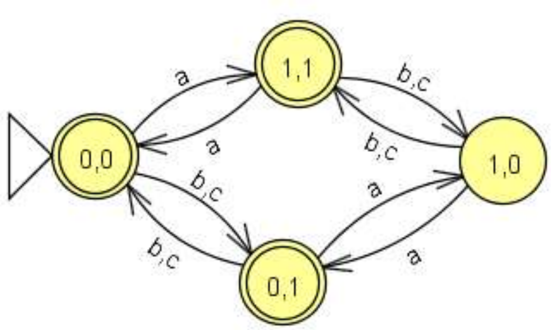
2. $L = \{w \in \{a, b, c\}^* : (|w| \text{ is even}) \rightarrow (w \text{ contains an even number of a's})\}$.

(a) Show that L is regular by using its definition and closure properties of regular languages.

This is equivalent to $\{w \in \{a, b, c\}^* : (|w| \text{ is odd}) \vee (w \text{ contains an even number of a's})\}$. Union of two regular languages is regular.

(b) Show that L is regular directly (***via*** a Finite State Machine or a regular expression) Based on part (a),

$(a \cup b \cup c)\ ((a \cup b \cup c)\ (a \cup b \cup c))^* \cup ((b\cup c)^*a(b\cup c)^*a(b\cup c)^*)^*$ or



Naming conventions for my states:

First number is the parity of the number of a's

Second number is the parity of the length of the string.

3. Use the Pumping theorem to show that the following language is not regular.

$L = \{a^i b^n : i, n > 0 \text{ and } i = n \text{ or } i = 2n\}$

Not regular. For any $k \geq 1$, let w be $a^k b^k$. w is in L. In the "pumping theorem breakup xyz, y must be a^p for some $p>0$. Choose $q=0$ to get $a^{k-p}b^k$, $k-p$ is neither i nor 2i, so this string is outside the language.

Note that it does NOT work to let w be $a^{2k}b^k$ and then choose $q=0$, since y could be a^k, and $a^{2k-k}b^k$ would still be in L.

4. Define *mid* to be a function on **strings**: For any string *s* over alphabet Σ:
   - If $|s| \leq 2$ then $mid(s) = \varepsilon$.
   - If $|s| > 2$ then let *x* and *z* be the single symbols from Σ that begin and end *s*. I.e. $s = xwz$ for some $w \in \Sigma^*$. Then $mid(s) = w$.

   For any **language** *L* over Σ, we define the function *midL*(*L*) as follows:
   $midL(L) = \{t \in \Sigma^* : t = mid(s) \text{ for some } s \in L\}$

(10) If L is the language denoted by the regular expression a*ba*b, what is *midL*(a*ba*b)?

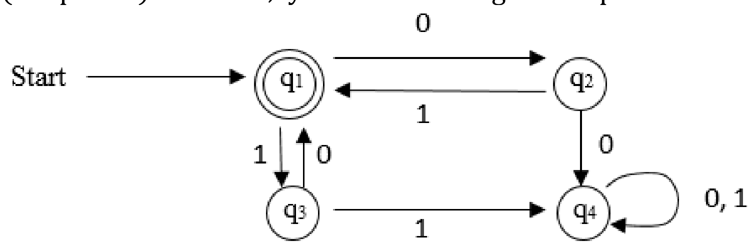
A representative element is $w = a^m b a^n b$. There are two cases.

If $m>0$, mid(w) is $a^{m-1}ba^n$. If $m=0$, mid(w) is a^n. So a regular expression for midL(L) is $a^*ba^* \cup a^*$

(15) Show that the set of non-regular languages is not closed under *midL*.
Hint: Consider $L = \{ba^pb : p > 0 \text{ and p prime}\} \cup \{ca^pc : p > 0 \text{ and p not prime}\}$.

For the given language, $midL(L) = \{a^p, p > 0 \text{ and p prime}\} \cup \{a^p, p > 0 \text{ and p not prime}\} = \{a\}^+$, which is regular.

5. (15 points) In HW6, you found a regular expression for the language accepted by the following DFSM:



From the HW 6 solutions, here are some of the r_{ijk} values from that calculation:

	k=0	k=1	k=2
r_{11k}	ε	ε	(01)*
r_{12k}	0	0	0(10)*
r_{13k}	1	1	(01)*1
r_{14k}	∅	∅	0(10)*0
r_{21k}	1	1	(10)*1
r_{22k}	ε	ε ∪ 10	(10)*
r_{23k}	∅	11	(10)*11
r_{24k}	0	0	0
r_{31k}	0	0	0(01)*
r_{32k}	∅	00	00(10)*
r_{33k}	ε	ε∪01	ε∪0(01)*1
r_{34k}	1	1	1
r_{41k}	∅	∅	∅
r_{42k}	∅	∅	∅
r_{43k}	∅	∅	∅
r_{44k}	ε ∪ 0 ∪ 1	ε ∪ 0 ∪ 1	ε ∪ 0 ∪ 1

The question:
What is r_{223}? If you correctly use the formula from class, you do not have to simplify your regular expression.

$r_{223} = r_{222} \cup r_{232} r_{332}^* r_{322}$

= (10)* ∪ (10)*11 (ε∪0(01)*1)* 00(10)*

6. (15) Define a decision procedure to answer the following question for DFSMs over Σ = {*a*, *b*}:
Given a DFSM *M*, does *L*(*M*) contain at least one string that begins with ab?

Your procedure may (and probably should) use (i.e. call) algorithms from the textbook and the homework.

1. Build a FSM M* that accepts the language denoted by regular expression *ab*(*a* ∪ *b*)*.
2. Build a FSM M** that accepts L(M) ∩ L(M*).
3. If L(M**) is empty, return *False*, else return *True*.