MA/CSSE 474 Exam 1 Winter 2013-14 Name________________________ Section: 02 (3rd)___ 03(4th) ___

Closed book and notes, except for one 8.5 x 11 sheet of paper (can be 2-sided).

Put your name on that paper and turn it in (in a separate pile) when you turn in your exam paper.

No electronic devices, especially you may not use anything with headphones or earbuds.

Scores:

Problem	Possible	Score
1	40	
2	15	
3	10	
4	10	
5	15	
6	10	
7	30 (18)	
Total	**130**	

1. (40 points) Circle T or F to indicate whether it is *True* or *False*. IDK means "I don't know"
If the statement is sometimes False, then False is the correct answer. **You need not give proofs or counterexamples.**
For each part, you earn **2 points** for circling IDK, **4** for circling the correct answer, **-2** for circling the incorrect answer, and **0** if you leave it blank. Leaving it blank is silly, since you get more points for IDK.

a) T F IDK Every alphabet is finite.

b) T F IDK The largest possible language is countably infinite.

c) T F IDK The set of all languages over alphabet {a} is countably infinite

d) T F IDK INF (the set of infinite languages) is closed under complement.
I.e. the complement of an infinite language is infinite.

e) T F IDK Every regular language is finite.

f) T F IDK The complement of a regular language must be regular.

g) T F IDK For every language L, $L\emptyset L \subseteq L$.

h) T F IDK For every language L, $L \subseteq L\emptyset L$

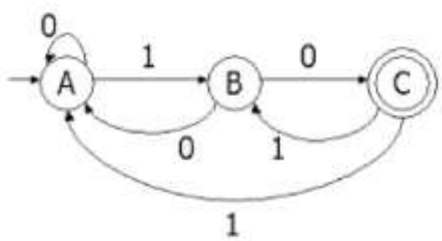
i) T F IDK The power set of $\emptyset$ is $\emptyset$.

j) T F IDK Recall that maxstring(L) = $\{w\in L: \forall z\in\Sigma^*(z\neq\varepsilon \rightarrow wz\notin L)\}$.
There is a language L such that maxstring(L) contains the empty string ε.

2. (15 points) Let $L\subseteq \Sigma^*$ for some alphabet Σ. In class we defined the relation $\approx_L$ on strings from Σ^* to be **$x \approx_L y$ if and only if $\forall z\in\Sigma^*(xz\in L \leftrightarrow yz\in L)$**. We showed that $\approx_L$ is an equivalence relation and used the notation [w] to mean "the equivalence class that contains w".

Carefully prove that for $\forall x,y\in\Sigma^*(\forall a\in\Sigma(([x] = [y]) \rightarrow ([xa] = [ya])))$. *This is NOT an induction proof.*

3. (10 points) The following nondeterministic finite state machine accepts which of the following strings? (Only one is correct).



- 1011101
- 1110100
- 1011010
- 00110100

4. (10 points) If we run the *ndsfsmtodfsm* algorithm on the above NDFSM, which of the following potential states of the resulting DFSM is *not* reachable?

- {B}
- {A}
- {A,B,C}
- {}

5. (15 points) Let $\Sigma = \{0, 1, 2\}$. Let L be $\{w \in \Sigma^* : \exists a,b \in \Sigma(\exists x,y \in \Sigma^*: w=x0ab1y\}$.
Draw the state diagram for a FSM M such that L = L(M).

6. (10 points) Let $\Sigma=\{a\}$, and let $C = \{L \subseteq \Sigma^* : (L=L(M) \text{ for some DFSM } M) \wedge (M=(K, \Sigma, s, A, \delta)) \wedge (|K|=2)\}$.

How many different languages are in C? ____

If you get the correct number, you earn full credit.
But feel free to list the languages, in case it helps you earn partial credit.

7. (30 points) For this induction problem, you have a choice. You can do the first one to earn up to 30 points.
Or you can do the simpler second one to earn up to 18 of the 30 possible points.
If you write something for both parts, be sure to indicate which one you want me to grade.
Whichever one you do, give the reasons for your steps, and in particular, make it clear where and how you use the induction hypothesis. *Continue your proof on the back of this page if necessary*.

We have seen two different notations for a series of transitions by a DFSM $M=(K, \Sigma, s, A, \delta)$.

On class day 1, we saw the "extended delta" function. Here, I will call that function $\hat{\delta}$ to distinguish it from δ, since they take arguments of different types:

$(\hat{\delta} : K \times \Sigma^* \to K$, while $\delta: K \times \Sigma \to K)$.

Here is the recursive definition of $\hat{\delta}$:

$\hat{\delta}(q, \varepsilon) = q$,

$\hat{\delta}(q, wa) = \delta(\hat{\delta}(q,w), a)$ for all $w \in \Sigma^*$, $a \in \Sigma$.

The textbook's $\vdash^*$ operator can be defined recursively:
For all $w,t \in \Sigma^*$, $a \in \Sigma$,

$(q, t) \vdash^* (q, t)$

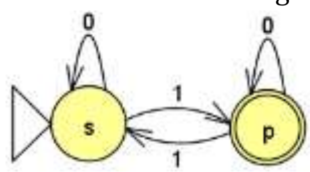
If $(q, wt) \vdash^* (p, t)$ for some $p \in K$ with $(p, a) \vdash (r, \varepsilon)$, then $(q, wat) \vdash^* (r, t)$

Prove (using induction on $|w|$ where needed)

$\forall q,r \in K, (\forall w \in \Sigma^* (\hat{\delta}(q,w) = r \leftrightarrow (q, w) \vdash^* (r, \varepsilon)))$.

I use the $\hat{\delta}$ notation (described in the other part of this problem, because I think it will make this proof easier to express. Feel free to use the textbook's $\vdash^*$ notation instead.

Consider the following DFSM:



Carefully prove, using mathematical induction on $|w|$ where needed, that $\hat{\delta}(s, w) = p$ if and only if w contains an odd number of 1's.