

Name: \_\_\_\_\_ KEY \_\_\_\_\_

Grade: \_\_\_\_\_ <-- instructor use

1. If we have a machine  $M(x, y)$  that multiplies two integers, how can we use it to make a machine that accepts **INTEGERPROD?**

Suppose we have a machine  $M(x,y)$  that multiplies two integers.

Given a string in the form  $\langle \text{int}_1 \rangle * \langle \text{int}_2 \rangle = \langle \text{int}_3 \rangle$

$X = \text{convertToInt}(\langle \text{int}_1 \rangle)$

$Y = \text{convertToInt}(\langle \text{int}_2 \rangle)$

$Z = \text{convertToInt}(\langle \text{int}_3 \rangle)$

If  $z = M(x,y)$  then accept. Else reject

2. If we have a machine that accepts INTEGERPROD, how can we use it to multiply two integers?

Suppose we have a program  $P$  that checks whether strings like the one above are in INTEGERPROD.

This program computes the product of two integers  $x$  and  $y$ .

Enumerate the strings that are encodings of natural numbers.

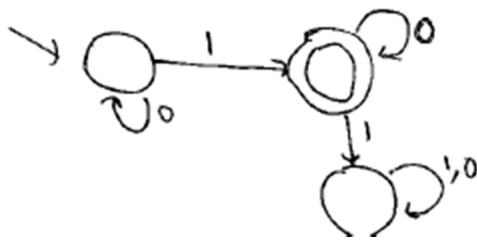
For each one,  $\langle z \rangle$ , feed the string  $\langle x \rangle * \langle y \rangle = \langle z \rangle$  to  $P$ .

If  $P$  accepts, return  $z$ . Otherwise, keep going.

$P$  will eventually accept

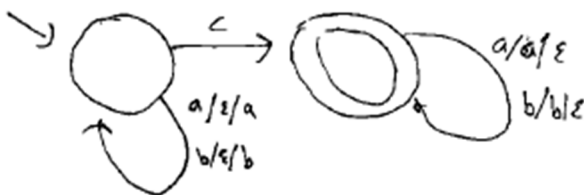
3. Draw the diagram for a FSM (finite-state machine) that recognizes the language

$L = \{ w \in \{0, 1\}^* : \exists n, k \in \mathbb{N} ( w = \langle n \rangle \wedge n = 2^k ) \}$  Where  $\langle n \rangle$  means the binary representation of  $n$ .



Students may have correct variations

4. Draw the diagram for a PDA (push-down automaton) that recognizes  $L = \{wcw^R : w \in \{a, b\}^*\}$ .



Must end with an empty stack in order to accept

lots of variations on this are possible

5. Describe (in English) the actions of a TM to recognize  $A^nB^nC^n$ .

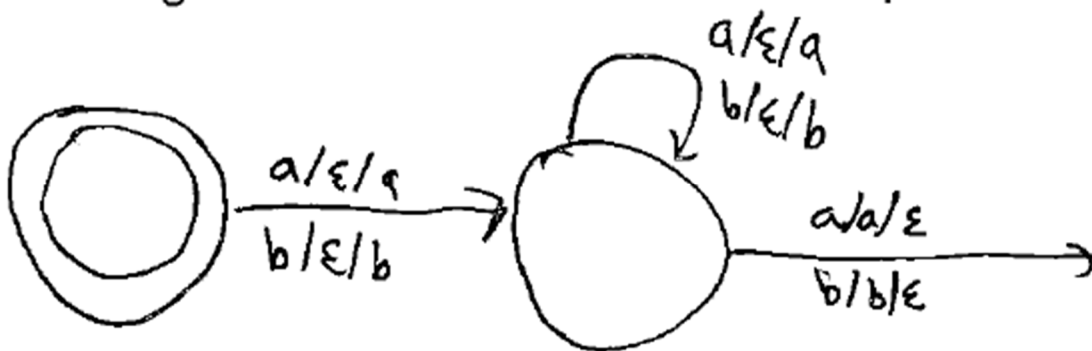
mark left and right ends.  
 If a, erase, then erase a b, then a c  
 move back to mark.  
 keep repeating until there is no a, or  
 b or c before a, or c before b.

Students may have their own variations

6. What does it mean for a language to be semidecidable?

$\exists$  a TM that accepts all strings in the language

7. Draw a diagram for a nondeterministic PDA to accept  $\text{PalEven} = \{ww^R : w \in \{a,b\}^*\}$



8. Tell your instructor about anything from today's session (or from the course so far) that you found confusing or still have a question about. If none, please write "None". Continue on the back if needed.