MA/CSSE 474 HW 14 problems (highlighted problems are the ones to turn in)

Be sure to read the numerous questions and answers in the main assignment document

13.13 (#1)

13.13d (#2) 6

13.14 (#3) 6,9

14.1a (#4) 6

14.1c (#5) 9

14.1d (#6)

14.1e (#7)

17.1a (#8) 6

17.1b (#9)

17.3a (#10) 9

17.3b-d (#11)

13. Are the context-free languages closed under each of the following functions? Prove your answer.

a. $chop(L) = \{w : \exists x \in L\ (x = x_1 c x_2 \wedge x_1 \in \Sigma_L{}^* \wedge x_2 \in \Sigma_L{}^* \wedge c \in \Sigma_L \wedge |x_1| = |x_2| \wedge w = x_1 x_2)\}$

b. $mix(L) = \{w : \exists x, y, z : (x \in L, x = yz, |y| = |z|, w = yz^R)\}$

c. $pref(L) = \{w : \exists x \in \Sigma^*(wx \in L)\}$

d. $middle(L) = \{x : \exists y, z \in \Sigma^*(yxz \in L)\}$

e. Letter substitution

f. $shuffle(L) = \{w : \exists x \in L$ (w is some permutation of x)$\}$

g. $copyreverse(L) = \{w : \exists x \in L\ (w = xx^R)\}$

14. Let $alt(L) = \{x : \exists y, n\ (y \in L, |y| = n, n > 0, y = a_1 \cdots a_n, \forall i \leq n\ (a_i \in \Sigma)$, and $x = a_1 a_3 a_5 \cdots a_k$, where $k =$ (if n is even then $n - 1$ else n))$\}$.

a. Consider $L = a^n b^n$. Clearly describe $L_1 = alt(L)$.

b. Are the context free languages closed under the function *alt*? Prove your answer.

13.14 Examples:

If L is the language denoted by r.e. (ab)*, then alt(L) is denoted by r.e. a*.

If L is the language denoted by r.e. (abcdefg)*, then alt(L) is denoted by r.e. (aceg)* .

If L is the language denoted by r.e. a or the language denoted by r.e. a*, then alt(L) is L.

1. Give a decision procedure to answer each of the following questions:

a. Given a regular expression α and a PDA M, is the language accepted by M a subset of the language generated by α?

b. Given a context-free grammar G and two strings s_1 and s_2, does G generate $s_1 s_2$?

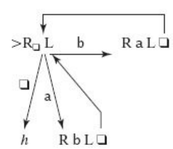
c. Given a context-free grammar G, does G generate at least three strings?

d. Given a context-free grammar G, does G generate any even length strings?

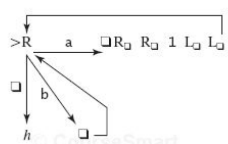
e. Given a regular grammar G, is $L(G)$ context-free?

1. Give a short English description of what each of these Turing machines does:

a. $\Sigma_M = \{a, b\}$. $M =$



b. $\Sigma_M = \{a, b\}$. $M =$



17.1a Don't just describe in English the *individual steps* the machine makes. Your answer should be a global one: in particular, describe how the final tape content is different from the original tape content.

3. Construct a standard, deterministic, one-tape Turing machine M to compute each of the following functions:

a. The function sub_3, which is defined as follows:

$$sub_3(n) = n - 3 \text{ if } n > 2$$
$$0 \text{ if } n \leq 2.$$

Specifically, compute sub_3 of a natural number represented in binary. For example, on input 10111, M should output 10100. On input 11101, M should output 11010. (*Hint*: You may want to define a subroutine.)

b. Addition of two binary natural numbers (as described in Example 17.13). Specifically, given the input string $<x>;<y>$, where $<x>$ is the binary encoding of a natural number x and $<y>$ is the binary encoding of a natural number y, M should output $<z>$, where z is the binary encoding of $x + y$. For example, on input 101;11, M should output 1000.

c. Multiplication of two unary numbers. Specifically, given the input string $<x>;<y>$, where $<x>$ is the unary encoding of a natural number x and $<y>$ is the unary encoding of a natural number y, M should output $<z>$, where z is the unary encoding of xy. For example, on input 111;1111, M should output 111111111111.

d. The proper subtraction function *monus*, which is defined as follows:

$$monus(n, m) = \begin{cases} n - m & \text{if } n > m \\ 0 & \text{if } n \leq m. \end{cases}$$

Specifically, compute *monus* of two natural numbers represented in binary. For example, on input 101;11, M should output 10. On input 11;101, M should output 0.

17.4
(#12) 9

4. Construct a Turing machine M that computes the function f: $\{\mathsf{a}, \mathsf{b}\}^* \rightarrow N$, where:

$$f(x) = \text{the unary encoding of } max(\#_{\mathsf{a}}(x), \#_{\mathsf{b}}(x)).$$

For example, on input aaaabb, M should output 1111. M may use more than one tape. It is not necessary to write the exact transition function for M. Describe it in clear English.

17.6
(#13) 3

6. Let M be a three-tape Turing machine with $\Sigma = \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$ and $\Gamma = \{\mathsf{a}, \mathsf{b}, \mathsf{c}, \square, 1, 2\}$. We want to build an equivalent one-tape Turing machine M' using the technique described in Section 17.3.1. How many symbols must there be in Γ'?