474 HW 10 problems (highlighted problems are the ones to turn in)

- 11.1, 2, 3 (#1, 2, 3)
- 11.4 (#4) (3)
- 11.6b (#5) (6)
- 11.6c (#6) (6)
- 11.6d (#7) (6)
- 11.6e (#8)
- (#9) not from book
- 11.6h (#10) (6)
- 11.6i (#11) (6)
- 11.6k (#12) (6)
- 11.8b (#13) (6)
- 11.9 (#14)

1. Let $\Sigma = \{a, b\}$. For the languages that are defined by each of the following grammars, do each of the following:

  i. List five strings that are in L.

  ii. List five strings that are not in L (or as many as there are, whichever is greater).

  iii. Describe L concisely. You can use regular expressions, expressions using variables (e.g., $a^n b^n$, or set theoretic expressions (e.g., $\{x: \dots\}$).

  iv. Indicate whether or not L is regular. Prove your answer.

  a. $S \rightarrow aS \mid Sb \mid \varepsilon$

  b. $S \rightarrow aSa \mid bSb \mid a \mid b$

  c. $S \rightarrow aS \mid bS \mid \varepsilon$

  d. $S \rightarrow aS \mid aSbS \mid \varepsilon$

2. Let G be the grammar of Example 11.12. Show a third parse tree that G can produce for the string (())().

3. Consider the following grammar G:

$$S \rightarrow 0S1 \mid SS \mid 10$$

Show a parse tree produced by G for each of the following strings:

  a. 010110.

  b. 00101101.

4. Consider the following context free grammar G:

$$S \rightarrow aSa$$
$$S \rightarrow T$$
$$S \rightarrow \varepsilon$$
$$T \rightarrow bT$$
$$T \rightarrow cT$$
$$T \rightarrow \varepsilon$$

One of these rules is redundant and could be removed without altering $L(G)$. Which one?

6. Show a context-free grammar for each of the following languages L:

  a. BalDelim = $\{w$: where w is a string of delimiters: (,), [,], {, }, that are properly balanced$\}$.

  b. $\{a^i b^j : 2i = 3j + 1\}$.

  c. $\{a^i b^j : 2i \neq 3j + 1\}$.

  d. $\{w \in \{a, b\}^* : \#_a(w) = 2 \cdot \#_b(w)\}$.}.

  e. $L = \{w \in \{a, b\}^* : w = w^R\}$.

  f. $\{a^i b^j c^k : i, j, k \geq 0 \text{ and } (i \neq j \text{ or } j \neq k)\}$.

  g. $\{a^i b^j c^k : i, j, k \geq 0 \text{ and } (k \leq i \text{ or } k \leq j)\}$.

  h. $\{w \in \{a, b\}^*$: every prefix of w has at least as many a's as b's$\}$.

  i. $\{a^n b^m : m \geq n, m\text{-}n \text{ is even}\}$.

  j. $\{a^m b^n c^p d^q : m, n, p, q \geq 0 \text{ and } m + n = p + q\}$.

  k. $\{xc^n : x \in \{a, b\}^* \text{ and } (\#_a(x) = n \text{ or } \#_b(x) = n)\}$.

  l. $\{b_i \# b_{i+1}{}^R : b_i$ is the binary representation of some integer i, $i \geq 0$, without leading zeros$\}$. (For example 101#011 $\in L$.)

  m. $\{x^R \# y : x, y \in \{0, 1\}^* \text{ and } x \text{ is a substring of } y\}$.

9. (t-6) Show the details of the definition of a CFG that generates $\{a^i b^j c^k : i, j, k \geq 0 \text{ and } (i + j = k)\}$

8. Consider the unambiguous expression grammar G' of Example 11.19.

  a. Trace a derivation of the string id+id*id*id in G'.

  b. Add exponentiation (**) and unary minus (−) to G', assigning the highest precedence to unary minus, followed by exponentiation, multiplication, and addition, in that order.

9. Let $L = \{w \in \{a, b, \cup, \varepsilon, (,), *, {}^+\}^* : w$ is a syntactically legal regular expression$\}$.

  a. Write an unambiguous context-free grammar that generates L. Your grammar should have a structure similar to the arithmetic expression grammar G' that we presented in Example 11.19. It should create parse trees that:

  - Associate left given operators of equal precedence, and
  - Correspond to assigning the following precedence levels to the operators (from highest to lowest):
    - * and +
    - concatenation
    - $\cup$

  b. Show the parse tree that your grammar will produce for the string $(a \cup b)$ ba*.