

Name: \_\_\_\_\_ Key \_\_\_\_\_

Grade: \_\_\_\_\_ &lt;-- instructor use

1. Why can't a PDA recognize the language  $A^nB^nC^n$ ?

A stack can be used to match the counts of two things, but not three.

2. Describe (in English) the actions of a TM that recognizes  $A^nB^nC^n$ .

mark left and right ends.  
 if a, erase, then erase a b, then a c  
 move back to mark.  
 keep repeating until there is no a, or  
 b or c before a, or c before b.

Students may have their own variations

3. What does it mean for a language to be *semidecidable*?

$\exists$  a TM that accepts all strings in the language

4. What is a decision problem?

A decision problem is one that has a yes/no answer for each instance.

5. Is the problem: "Are there any prime Fermat numbers greater than 1,000,000?" decidable? Explain.

Yes. One of the following algorithms is correct for this problem.

- def t(): return TRUE
- def f(): return FALSE

So there is a decision procedure, we just don't know which one it is.

6. If  $L$  is  $\{b^na : n \geq 0\}$  What is  $\text{maxstring}(L)$ ? It is the same as  $L$ .

7. What does  $(q, w) \vdash_M (q', w')$  mean? The machine  $M$ , in state  $q$  with  $w$  as the remaining input, transitions to state  $q'$  with  $w'$  as the remaining input. This happens iff  $w = aw'$  for some  $a \in \Sigma$ , and  $\delta(q, a) = q'$ .

8. Prove: Every DFSA  $M$ , in configuration  $(q, w)$ , halts after  $|w|$  steps .

**Base case:** If  $w$  is  $\epsilon$ , it halts in 0 steps.

**Induction step:** Assume true for strings of length  $n$  and show for strings of length  $n+1$ .

Let  $w \in \Sigma^*$ ,  $|w| = n+1$  for some  $n \in \mathbb{N}$ .

Then  $w$  is  $ax$  for some  $a \in \Sigma$ ,  $x \in \Sigma^*$ ,  $|x| = n$ .

Let  $q'$  be  $\delta(q, a)$ . Then  $(q, w) \vdash_M (q', x)$

By induction, from configuration  $(q', x)$ ,  $M$  halts in  $n$  steps.

So, starting from the original configuration,  $M$  halts in  $n+1$  steps.

9. Draw the transition diagram (or transition table) for a DFSA that accepts  
 OddParity =  $\{w \in \{0, 1\}^* : w \text{ contains an odd number of 1s}\}$

