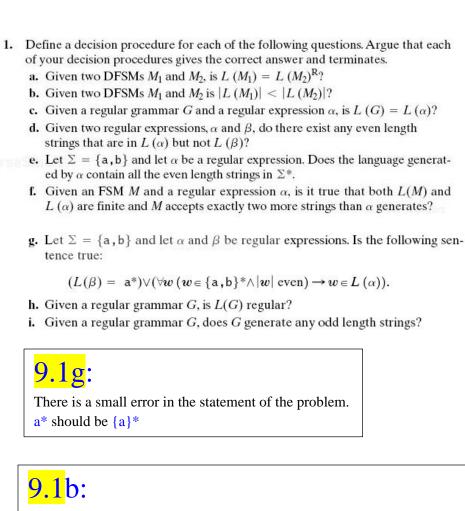
## 474 HW 09 problems (highlighted problems are the ones to turn in)

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8.8(#1)	
	7. Prove that the regular languages are closed under each of the following operations:
<mark>8.8 d, e</mark>	<b>a.</b> $pref(L) = \{w : \exists x \in \Sigma^*(wx \in L)\}.$
(#2) <mark>( 3, 6)</mark>	<b>b.</b> $suff(L) = \{w : \exists x \in \Sigma^*(xw \in L)\}.$
	<b>c.</b> $reverse(L) = \{x \in \Sigma^* : x = w^{\mathbb{R}} \text{ for some } w \in L\}.$
	<b>d.</b> letter substitution (as defined in Section 8.3).
	8. Using the definitions of <i>maxstring</i> and <i>mix</i> given in Section 8.6, give a precise def-
<mark>8.9</mark>	inition of each of the following languages:
(#3) ( <mark>9</mark> )	
	<b>a.</b> $maxstring(A^n B^n)$ .
	<b>b.</b> $maxstring(a^i b^j c^k, 1 \le k \le j \le i).$
	<b>c.</b> maxstring( $L_1L_2$ ), where $L_1 = \{w \in \{a, b\}^* : w \text{ contains exactly one } a\}$ and
	$L_2 = \{\mathbf{a}\}.$
<mark>8.10a</mark>	d. $mix((aba)^*)$ .
(#4) (9)	<b>e.</b> $mix(a*b*)$ .
	9. Prove that the regular languages are not closed under mix. a DFSM M = (K, $\Sigma$ , $\delta$ ,
	s, A) such that
	L(M)=L, construct a
<mark>8.16a</mark>	Definitions of <i>maxstring</i> and <i>mix</i> are on pages 181-182. DFSM
(#5) ( <mark>9</mark> )	$M^* = (K^*, \Sigma, \Delta^*, s^*, A^*)$
	10. Recall that $maxstring(L) = \{w : w \in L \text{ and } \forall z \in \Sigma^* (z \neq \varepsilon \rightarrow wz \notin L)\}.$ such that
8.16b	a. Prove that the regular languages are closed under masstring. $L(M^*)$ =maxstring(L).
(#6)	b. If $maxstring(L)$ is regular, must L also be regular? Prove your answer.
8.21	16. Define two integers i and j to be <i>twin primes</i> $\square$ iff both i and j are prime and $ i-2 $
(#7)	j-i  = 2.
	a. Let $L = \{w \in \{1\}^* : w \text{ is the unary notation for a natural number } n \text{ such that there exists a pair } p \text{ and } q \text{ of twin primes, both } > n.\}$ Is L regular?
	<b>b.</b> Let $L = \{x, y : x  is the decimal encoding of a positive integer i, y is the deci-$
	mal encoding of a positive integer $j$ , and $i$ and $j$ are twin primes}. Is L regular?
	indicided and bosidive integer f, and valid fare (with printes). Is E regard,
	21. For each of the following claims, state whether it is True or False. Prove your
	answer.
	<b>a.</b> There are uncountably many non-regular languages over $\Sigma = \{a, b\}$ .
	b. The union of an infinite number of regular languages must be regular.
<mark>8.21n</mark>	c. The union of an infinite number of regular languages is never regular.
(#8) ( <mark>12</mark> )	<b>d.</b> If $L_1$ and $L_2$ are not regular languages, then $L_1 \cup L_2$ is not regular.
	e. If $L_1$ and $L_2$ are regular languages, then $L_1 \otimes L_2 = \{w : w \in (L_1 - L_2) \text{ or } w \in (L_2 - L_1)\}$ is regular.
<mark>8.210</mark>	f. If $L_1$ and $L_2$ are regular languages and $L_1 \subseteq L \subseteq L_2$ , then L must be regular.
	<b>g.</b> The intersection of a regular language and a nonregular language must be
(#9) ( <mark>3</mark> )	regular.
	h. The intersection of a regular language and a nonregular language must not be
	regular.
	i. The intersection of two nonregular languages must not be regular.
	j. The intersection of a finite number of nonregular languages must not be regular.
	<b>k.</b> The intersection of an infinite number of regular languages must be regular.
	<b>I.</b> It is possible that the concatenation of two nonregular languages is regular.
	m. It is possible that the union of a regular language and a nonregular language
	is regular.

- n. Every nonregular language can be described as the intersection of an infinite number of regular languages.
- o. If L is a language that is not regular, then  $L^*$  is not regular.



9.1 (#10)

(#11) (<mark>6</mark>)

(#12) (<mark>6</mark>)

<mark>9.1b</mark>

9.1d

9.1g See note

below

<mark>9.1i</mark>

(#13) (<mark>6</mark>)

(#14) (<mark>6</mark>)

Note that |L(M)| means "the number of elements in the language accepted by the machine M. Note that for some machines M, the language is countably infinite.