474 HW 6 problems (highlighted problems are the ones to turn in)

6.1
1
6.1a
2 (3)

6.2 a-k,
n-p
3

6.2a
4 (3)

6.2c
5 (3)

6.2e
6 (3)

6.2h
7 (3)

6.3abe
8 (3-3-3)

6.4abd
9 (3-3-3)

1. Describe in English, as briefly as possible, the language defined by each of these regular expressions:
   - **a.** (b ∪ ba) (b ∪ a)* (ab ∪ b).
   - **b.** (((a*b*)*ab) ∪ ((a*b*)*ba))(b ∪ a)*.
2. Write a regular expressions to describe each of the following languages:
   - **a.** $\{w \in \{a, b\}^* :$ every a in w is immediately preceded and followed by b$\}$.
   - **b.** $\{w \in \{a, b\}^* : w$ does not end in ba$\}$.
   - **c.** $\{w \in \{0, 1\}^* : \exists y \in \{0, 1\}^* \, (|xy|$ is even$)\}$.
   - **d.** $\{w \in \{0, 1\}^* : w$ corresponds to the binary encoding, without leading 0s, of natural numbers that are evenly divisible by 4$\}$.
   - **e.** $\{w \in \{0, 1\}^* : w$ corresponds to the binary encoding, without leading 0s, of natural numbers that are powers of 4$\}$.
   - **f.** $\{w \in \{0\text{-}9\}^* : w$ corresponds to the decimal encoding, without leading 0s, of an odd natural number$\}$.
   - **g.** $\{w \in \{0, 1\}^* : w$ has 001 as a substring$\}$.
   - **h.** $\{w \in \{0, 1\}^* : w$ does not have 001 as a substring$\}$.
   - **i.** $\{w \in \{a, b\}^* : w$ has bba as a substring$\}$.
   - **j.** $\{w \in \{a, b\}^* : w$ has both aa and bb as substrings$\}$.
   - **k.** $\{w \in \{a, b\}^* : w$ has both aa and aba as substrings$\}$.
   - **l.** $\{w \in \{a, b\}^* : w$ contains at least two b's that are not followed by an a$\}$.
   - **m.** $\{w \in \{0, 1\}^* : w$ has at most one pair of consecutive 0s and at most one pair of consecutive 1s$\}$.
   - **n.** $\{w \in \{0, 1\}^* :$ none of the prefixes of w ends in 0$\}$.
   - **o.** $\{w \in \{a, b\}^* : \#_a(w) \equiv_3 0\}$.
   - **p.** $\{w \in \{a, b\}^* : \#_a(w) \leq 3\}$.
   - **q.** $\{w \in \{a, b\}^* : w$ contains exactly two occurrences of the substring aa$\}$.
   - **r.** $\{w \in \{a, b\}^* : w$ contains no more than two occurrences of the substring aa$\}$.
   - **s.** $\{w \in \{a, b\}^* - L\}$, where $L = \{w \in \{a, b\}^* : w$ contains bba as a substring$\}$.
   - **t.** $\{w \in \{0, 1\}^* :$ every odd length string in L begins with 11$\}$.
   - **u.** $\{w \in \{0\text{-}9\}^* : w$ represents the decimal encoding of an odd natural number without leading 0s.
   - **v.** $L_1 - L_2$, where $L_1 =$ a*b*c* and $L_2 =$ c*b*a*.
   - **w.** The set of legal United States zip codes.
   - **x.** The set of strings that correspond to domestic telephone numbers in your country.
3. Simplify each of the following regular expressions:
   - **a.** (a ∪ b)* (a ∪ ε) b*.
   - **b.** (∅* ∪ b) b*.
   - **c.** (a ∪ b)*a* ∪ b.
   - **d.** ((a ∪ b)*)*.
   - **e.** ((a ∪ b)$^+$)*.
   - **f.** a ((a ∪ b)(b ∪ a))* ∪ a ((a ∪ b) a)* ∪ a ((b ∪ a) b)*.
4. For each of the following expressions E, answer the following three questions and prove your answer:
   - i. Is E a regular expression?
   - ii. If E is a regular expression, give a simpler regular expression.
   - iii. Does E describe a regular language?
   - **a.** ((a ∪ b) ∪ (ab))*.
   - **b.** (a$^+$ a^{n}b^n).
   - **c.** ((ab)*∅).
   - **d.** (((ab) ∪ c)* ∩ (b ∪ c*)).
   - **e.** (∅* ∪ (bb*)).

6.2: For the "to-turn-in" parts of 6.2, aim for as simple a regular expression as you can come up with. Some of the credit may be for simplicity. If your expression is very complicated, some annotation may help the grader to know whether it is correct. The "burden of correctness" is on you. I did not require some of the more complex parts of this problem due to difficulties with grading, but you should try some of them.

Note for part c: The x is a typo, should be w. $|wy|$ means "the length of string wy"

Continued on next page

6.5
10(6-6)

5. Let $L = \{\mathrm{a}^n\mathrm{b}^n : 0 \leq n \leq 4\}$.

a. Show a regular expression for L.

b. Show an FSM that accepts L.