

473: Notes on Day 5 slides:

Slide 3 Asymptotic Analysis Example:

$C < 1$. The infinite series converges to $1/(1-c)$, so $\Theta(1)$

$C = 1$ $f(n) = n$, so it is $\Theta(n)$

$C > 1$ $f(n) = (c^{n+1} - 1) / (c - 1)$. The limit of $f(n)$ divided by c^n as $n \rightarrow \infty$ is $c/(c-1)$,
so $f(n)$ is $\Theta(c^n)$.

Slide 7: A Creative $O(\log N)$ Algorithm?

The file **fib3.py** has the solution, and **fib3-incomplete.py** has it partially completed.

```
#Solution for matrix_power:
def matrix_power(m, n):
    result = identity_matrix
    power = x
    while n > 0:
        if n % 2 == 1:
            result = matrix_multiply(result, power)
            power = matrix_multiply(power, power)
        # print "In loop:", n, power, result
        n = n / 2
    return result
```

Slide 8: Reconsider our Fibonacci algorithms

Note on recursive: The length of $F(n)$ is $.694n$, which is $O(n)$.

Note on last one: The # of bits in the numbers in the matrix at most doubles with each matrix multiplication.

Thus we get AN UPPER BOUND OF $M(1) + M(2) + M(4) + M(8) + \dots + M(F(n))$

If $a = 2$, we get (Maple notation):

```
> sum((2^i)^2, i=0..log[2](n));
```

```
> simplify(sum((2^i)^log[2](3), i=0..log[2](n)));
```

Slide 12: Modular Addition and Multiplication

0 to $N - 1$, 0 to $2(N-1)$, $\Theta(n)$.

0 to $(N-1)^2$, $2n$ $\Theta(n^2)$

Slide 14: Modular Exponentiation

To do better, we can use an algorithm like our previous recursive exponentiation algorithm

