

### 473: Instructor Notes from Day 11 slides:

Slide 4: Example (from Wikipedia)

Another example (Dasgupta p 33):

$$N=55 = 5 \cdot 11. \quad e = 3, \quad d = 3^{-1} \pmod{40} = 27.$$

$$13^3 = 52 \pmod{55}, \quad 52^{27} = 13 \pmod{55}.$$

Slide 10: Proof of the property

$x^{p-1} \equiv 1 \pmod{p}$ , so any power of that (in particular  $x^{k(p-1)(q-1)} \equiv 1 \pmod{p}$ ).

Thus  $x^{1+k(p-1)(q-1)} \equiv x \pmod{p}$ .

Because  $p$  and  $q$  are prime, if  $p$  and  $q$  divide a number, so does  $pq$ .

Slide 11: RSA Security

We'd have to find  $d$ . This would be easy if we knew  $p$  and  $q$ , but factoring is hard.

ON BOARD: Note that  $a^{N-1} = a^{u(2^t)} = (\dots ((a^u)^2) \dots)^2$  square  $t$  times

Slide 12: Amortized efficiency analysis

To [decrease](#) an [amount gradually](#) or in [installments](#), especially in order to [write off](#) an [expenditure](#) or [liquidate](#) a [debt](#). *He obtained a mortgage with the interest payments **amortized** over the life of the loan.*

to write off a cost of (an asset) gradually.

Slide 13: Growable Array (implement ArrayList)

$$12+13+14+15+\dots+N = N(N+1)/2 - 11(12)/2. \quad \text{Amortized } \Theta(N)$$

Do it only for the case

$$N = 12 \cdot 2^k + 1. \quad 12(1 + 2 + 4 + 2^k) = 12(2^{k+1} - 1) = 2(N-1) - 12 = 2N - 14. \quad \text{Amortized } \Theta(1)$$