MA/CSSE 473 – Design and Analysis of Algorithms **In-class Quiz Day 06**

Name:_____Solution_____ Score:____/ 9 circle your Section # 01(3rd) 02 (4th)

1. If c is a positive constant, find a simple big-Theta expression (as a function of n) for the following sum

when 0 < c < 1

The infinite series converges to 1/(1-c), so Θ(1)

when c = 1

f(n) = n, so it is Θ(n)

when c > 1

f(n) = (c^(n+1) – 1) / (c – 1). The limit of f(n) divided by c^n as n→∞ is c/(c-1),
so f(n) is Θ(c^n).

2. Which is harder (computationally): factoring numbers or determining whether numbers are prime?
factoring is harder

3. Trace the integer division algorithm from class for `divide(19, 4)`.

divide(19, 4):
q, r = divide(9, 4)
q, r = divide (4, 4)
q, r = divide(2, 4)
q, r = divide(1, 4)
q, r = divide(0, 4) = 0, 0
x is odd, so
r = 1
return 0, 1
q, r, = 0, 2
return 0, 2
q, r = 0, 4
q, r = 1, 0
return 1, 0
q, r, = 2, 0
x is odd, so
q, r = 2, 1
return 2, 1
q, r = 4, 2
x is odd, so
q, r, = 4, 3
return 4, 3

4. When exponentiating n-bit numbers x^y (mod N), where N is also n-bit, how many recursive calls are needed? **n**

5. Each call is Θ(**n**)

6. Entire exponentiation algorithm is Θ($\mathbf{n^2}$)

7. Prove that there is always a survivor in an odd pie fight.

Base case. P(1), 3 people. Suppose A and B are the closest pair, and C is the third person. Since all of the distances are different, the distances between A and C, and between B and C are strictly larger than the distance between A and B. Thus A and B throw pies at each other, and C is the survivor.
Induction step. Assume that P(k) is true (in every pie fight with 2k+1 people, there is a survivor). Show that P(k+1) is true (In every pie fight with 2K+3 people, there is a survivor).
Let A and B be the closest pair in this group of 2k+3 people. They throw pies at each other.
If someone else throws a pie at one of them, then for the remaining 2k+1 people, there are only 2k pies to hit them, so somene survives.
If no one else throws a pie at A or B, then the other people comprise a 2k+1 pie fight, which has a survivor, by the induction assumption.

8. Tell me about anything from today's lecture that you found confusing or feel that we need to spend more time on. Be as specific as you can, (or write N/A).

Any answer is fine, but they must have an answer (or N/A)

9. What questions do you have (from today's lecture, from the reading, or from the course in general)? Or write N/A.

Any answer is fine, but they must have an answer (or N/A)