

MA/CSSE 473 Day 18 Some "left-over" divide and conquer algorithms:

1. Fake Coin problem: How many weighings are necessary to find the lighter coin (assume there is exactly one lighter coin) if we use a decrease by a factor of 2 strategy?
2. Median-finding (use a quicksort-like partition, often called quickselect)
3. Explain the winning strategy for two-player one-pile Nim where a player can take  $1 \dots m$  chips on one turn, and the winner is the player who takes to take the last chip.
4. What is the winning strategy for 2-pile Nim? Each player can take any nonzero number of chips *from either of the piles*.

**A strategy for n-pile Nim.**

5. Define  $x \oplus y$ , the "Nim sum" of  $x$  and  $y$ . (note that  $\oplus$  is associative and commutative)
6. What is  $11 \oplus 14$ ? \_\_\_\_\_

7. Notation for  $x_i$ ,  $y_i$ ,  $s$ , and  $t$ :

8. Lemma 1 and its proof

9. Lemma 2 and its proof

10. Lemma 3 and its proof

11. Briefly describe the Josephus problem