MA/CSSE 473 – Design and Analysis of Algorithms

Homework 2 – (11 problems, 82 points total) Updated for Summer, 2017

These are to be turned in to a drop box on Moodle. You may write your solutions by hand and scan them if you wish. When a problem is listed by number, it is from Levitin. 1.1.2 means "Exercise 2 from section 1.1" .

Timing suggestion: This is a very long assignment; HW 1 is not quite as long. I recommend that you do part of this assignment *before* the due date for HW1. Because some students will take a while to "get into" this course, I chose to make HW 1 shorter and HW2 longer. If you want to make the workload more even, schedule you time pretending that a couple of HW2 problems are really in HW1.

Problems for enlightenment/practice/review (not to turn in, but you should at least think about them): How many of them you need to do serious work on depends on you and your background. I do not want to make everyone do one of them for the sake of the (possibly) few who need it. You can hopefully figure out which ones you need to do.

2.1.7 [2.1.7] (and 2.1.8. Effect of changing problem size on runtime)
2.1.10a [2.1.10] (chess-board doubling)
2.2.1 [2.2.1] (efficiency of sequential search)
2.2.2 [2.2.2] (informal definitions of asymptotic notations)
2.2.6 [2.2.6] (orders of growth for polynomials and exponentials)
2.2.9 [2.2.9] (effect of presorting on running time)
2.3.1 [2.3.1] (summation practice)
2.3.5 [2.3.5] (Secret algorithm)
2.3.6 [2.3.6] (Enigma algorithm)
2.3.12 [2.3.11] (von Neumann's Neighborhood)
2.3.13 [not in 2nd ed] (Page numbering).

13. *Page numbering* Find the total number of decimal digits needed for numbering pages in a book of 1000 pages. Assume that the pages are numbered consecutively starting with 1.

2.4.13[not in 2nd ed] (Frying Hamburgers)

13. *Frying hamburgers* There are n hamburgers to be fried on a small grill that can hold only two hamburgers at a time. Each hamburger has to be fried on both sides; frying one side of a hamburger takes one minute, regardless of whether one or two hamburgers are fried at the same time. Consider the following recursive algorithm for executing this task. If $n \leq 2$, fry the hamburger (or the two hamburgers together if $n = 2$) on each side. If $n > 2$, fry two hamburgers together on each side and then fry the remaining $n - 2$ hamburgers by the same algorithm.

a. Set up and solve the recurrence for the amount of time this algorithm needs to fry n hamburgers.

b. Explain why this algorithm does *not* fry the hamburgers in the minimum amount of time for all $n > 0$.

c. Give a correct recursive algorithm that executes the task in the minimum amount of time for all $n > 0$ and find a closed-form formula for the minimum amount of time.

2.5.3 [2.5.4] (Climbing stairs)
Another good practice problem to prepare for this assignment: The "growable array" exercise from 230. See the three files from days 01 and 02 in the 230-materials folder.

Problems to write up and turn in:

1. 2.1.4 (6 points) (socks and gloves) Note that a “selection”does not mean choosing a single glove. It means choosing all of the gloves or socks that you are going to choose. You don’t look at any of them until you havre chosen all of them.

   Previous questions and answers from Piazza:

   Instructor note: I am sure that Mr. Levitin assumes a fundamental difference between socks and gloves. In a pair of gloves there is a left glove and a right glove; in a pair of socks, there is no left-right distinction. I recognize that some high-end brands of socks, including the wonderful Keen socks that I am wearing right now, have distinct left and right socks. But that is not the norm, so you should assume that they are the same.

2. 2.1.5 (9 points) (number of digits in the representation of a positive integer)
   Note that there are four parts of this problem. If you have the 2nd edition of Levitin, see the "Problems" document. Points: (3, 3, 1, 2)

   Previous questions and answers from Piazza:

   Q on part d: I What is this question asking? What does it mean "in an accepted analysis framework" and what does it mean by n's size? Is it asking that it does not matter which base we use for n?
   A: "Accepted analysis framework" is what Section 2.1 of the textbook is about. If you haven't read it yet, you should do so before you do this problem. It is summarized on page 50 [50]. I think after reading those 5 or so pages, you'll know what the author means.

   n's size is just a way to say **n** with emphasis on how big n is.

   The problem is asking you to explain why it doesn't matter which base we use for our representation of n. n itself is independent of any base.

   It's ***the way we represent n on a computer or on paper*** that depends on a particular base.

   In CSSE 230 and CSSE 304, we talk a lot about the difference between an element of an Abstract Data Type and its representation. The same element can have many different representations, just as the same number can be expressed in many different bases. Our problem with grasping this is partly that since we normally use only one representation (base 10) in our minds we tend to merge the number itself with its representation. We think "38 is the number that comes after 37". More precise is "38 is the base 10 representation of the number that comes after the number whose base 10 representation is 37.

3. 2.2.3 (10 points) (big-theta of specific functions with proofs)
   For parts a&b, use limits;
   for e, use formal definitions of O and Ω;
   you should probably find specific values for the c and n_0 in the formulas on pages 53-55.
   for c & d, you can use the theorem on p 56.

4. 2.2.7a,d (4 points) (proof or disproof of properties using the formal definition)

5. 2.2.12 [2.2.10] (6 points) (door in a wall). Show that your algorithm is actually O(N).

6. 2.3.2 (8 points) (big-theta for various summations).

7. 2.3.11 [2.3.10] (10 points) (GE Algorithm – yeah, it's a big secret what GE stands for ☺) Include a quantitative indication of how much time is gained by removing the glaring inefficiency. For part a, you must (by hand) completely evaluate the summation before making any approximations. I see part (a) mainly as an exercise in evaluating nested sums.

8. 2.4.14 [5.3.10] (10 points) (Celebrity identification) **be sure to start this one early!** "Efficient" in this case means making the number of questions for N people as small as you can, and you should say how many questions (as a function of N) are required by your approach in the worst case.

Previous questions and answers from Piazza:

Q on part d: when it says you can only ask a person "do you know him/her?" does that refer to that person knowing the celebrity or someone else in the group of n people?
A: You can ask person A, "do you know person B?", not "do you know who the celebrity is?" or anything like that.

Also, "you" are an outside observer, not one pf the people in the room. So you only know what you learn by asking people in the room the question. Your goal is to find out whether there is a celebrity on the room and if so, who that celebrity is, using the minimum number of questions that is guaranteed to give you the correct answer.

9. Master Theorem Proof (8 points). These questions refer to the proof of the Master Theorem in Weiss section 7.5.3 (available on Moodle). **Later we will have homework problems that *use* the Master Theorem**.

This proof is sometimes part of CSSE 230, but it happens during the time of the big team project, so many students don't focus on it enough to really understand it. Here is your chance to do so! If you do not already feel comfortable with telescoping as a method of solving recurrence relations, read the early parts of section 7.5.3 carefully. If you do, you can start with the page before the proof. Levitin also has a proof of a stronger version of the Master Theorem in Appendix B.

You'll probably find one or more of the formulas from Levitin's Appendix A to be helpful here.

(a) (5 points) (7.11) Why is it $O(A^M)$? Why is the second equation true ($= O(A^M) = O(N^{\log_B A})$.)?
(b) (3 points) Sentence after (7.11). Why does the sum contain that many terms? Why does $A = B^k$ imply $A^M = N^k$?

Previous questions and answers from Piazza:

Q: (b) Why does the sum contain that many terms? **A:** This refers to the number of terms in the summation for equation 7.10. The sentence this question refers to says the sum contains $1+\log_b N$ terms. Why does Weiss say there are that many terms in equation 7.10?

10. Dasgupta questions (6 points, 2 for each part). Refers to an excerpt from Dasgupta's book (on Moodle)
(a) What does Dasgupta say are the two main ideas that changed the world? Do you agree? What else might you include in the list?

(b)Why is the simple Fibonacci algorithm at the bottom of page 4 (of the Dasgupta book) actually not O(n)?
(c) Show how to use al-Khwarizmi's technique to multiply 9 (first column) by 15 (second column).

Note on problem 10. These are easy “did you read this?” questions. On later assignments, there will be "real" problems based on the Dasgupta excerpt.

11. 3.1.7(b) (5 points) Stack of fake coins. Minimum # of weighings. Explain your answer.
[Not in 2nd edition. See the problems and hints document for details].