MA/CSSE 473 Day 20

Finish Josephus
Transform and conquer
Gaussian Elimination
LU-decomposition
AVL Tree
Maximum height
2-3 Trees
Student questions?

Recap: Josephus Problem

- n people, numbered 1-n, are in a circle
- Count starts with 1
- Every 2nd person is eliminated
- The last person left, J(n), is the winner
- Examples: n=8, n=7
- J(1) = 1
- Solution if n is even: J(2k) = 2*J(k) - 1
- Solution if n is odd: J(2k+1) = 2*J(k) + 1
- Use it to find J(2) … J(8)
- Clever solution: cyclic bit shift left

Transform and Conquer Algorithms

- Transform a problem to a simpler instance of the same problem – **instance simplification**
- Transformation to a different representation of the same instance – **representation change**
- Transformation to an instance of a different problem that we know how to solve – **problem reduction**

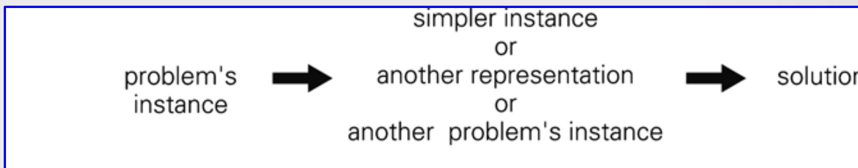

FIGURE 6.1 Transform-and-conquer strategy

Instance simplification: Presorting an Array

- The following problems are simplified by pre-sorting the array:
  - **Search** (can do Binary or Interpolation search)
  - Determine whether the array contains **duplicates**
  - Find the **median** of the array
  - Find the **mode** of the elements of the array
    - The most frequently-occurring element
  - A related problem: Anagrams
    - In a large collection of words, find words that are anagrams of each other
    - How can pre-sorting help?
    - Sort the letters of each word
  - Interval union problem from early part of PLC course

Instance Simplification: Gaussian Elimination (hopefully you saw basics in a DE or lin. Alg. course)

- Solve a system of n linear equations in n unknowns
  - Represent the system by an augmented coefficient matrix
  - Transform the matrix to triangular matrix by a combination of the following solution-preserving elementary operations:
    - exchange two rows
    - multiply a row by a nonzero constant
    - replace a row by that row plus or minus a constant multiple of a different row
  - Look at the algorithm and analysis on pp 207-208; if you can't understand them, ask in my office.
  - $\Theta(n^3)$ [previous HW problem]

Other Applications of G.E.

- Matrix inverse
  - Augment a square matrix by the identity matrix
  - Perform elementary operations until the original matrix is the identity.
  - The "augmented part" will be the inverse
  - More details and an example at http://en.wikipedia.org/wiki/Gauss-Jordan_elimination

Other Applications of G.E.

- Determinant calculation
  - Calculation of the determinant of a triangular matrix is easy
- What effect does each of the elementary operations have on the determinant value?
  - exchange two rows
  - multiply a row by a nonzero constant
  - replace a row by that row plus or minus a constant multiple of a different row
- Do these operations until you get a triangular matrix
- Keep track of the operations' cumulative effect on the determinant

LU Decomposition

- This can speed up all three applications of Gaussian Elimination
- Write the matrix A as a product of a Lower Triangular matrix L and an upper Triangular matrix U.
- Example:

$$[A]=\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

Splitting A up into L and U is an n^3 operation. https://rosettacode.org/wiki/LU_decomposition

$$[L]=\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \quad [U]=\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

Review: Representation change: AVL Trees (what you should remember…)

- Named for authors of original paper, **A**delson-**V**elskii and **L**andis (1962).
- An AVL tree is a height-balanced Binary Search Tree.
- A BST T is **height balanced** if T is empty, or if
  - | height(T_L) - height(T_R) | $\leq$ 1, and
  - T_L and T_R are both height-balanced.
- Show: Maximum height of an AVL tree with N nodes is Θ(log N) **Let's review that together**
- How do we maintain balance after insertion?
- **Exercise for later:** Given a pointer to the root of an AVL tree with N nodes, find the height of the tree in log N time
- Details on balance codes and various rotations are in the CSSE 230 slides that are linked from the schedule page.

Representation change: 2-3 trees

- Another approach to balanced trees
- Keeps all leaves on the same level
- Some non-leaf nodes have 2 keys and 3 subtrees
- Others are regular binary nodes.

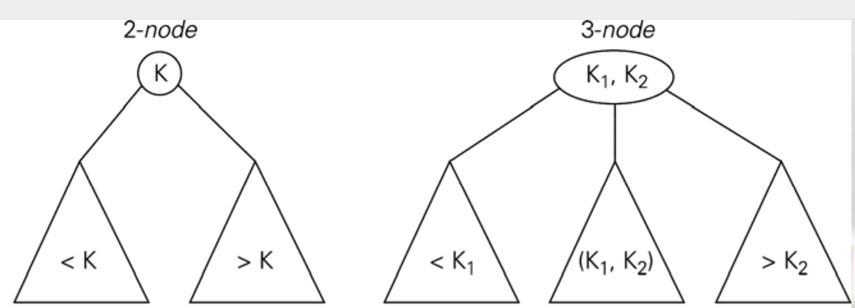

FIGURE 6.7 Two kinds of nodes of a 2-3 tree

2-3 tree insertion example

- More examples of insertion:
  http://www.cs.ucr.edu/cs14/cs14_06win/slides/2-3_trees_covered.pdf
  http://slady.net/java/bt/view.php?w=450&h=300

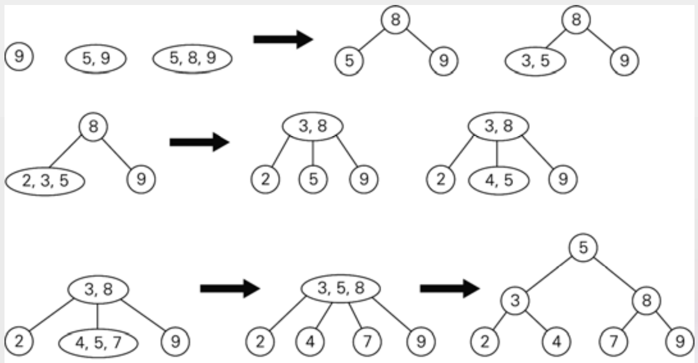

FIGURE 6.8 Construction of a 2-3 tree for the list 9, 5, 8, 3, 2, 4, 7

Add 10, 11, 12, … to the last tree

Efficiency of 2-3 tree insertion

- Upper and lower bounds on height of a tree with n elements?
- Worst case insertion and lookup times is proportional to the height of the tree.

2-3 Tree insertion practice

- Insert 84 into this tree and show the resulting tree