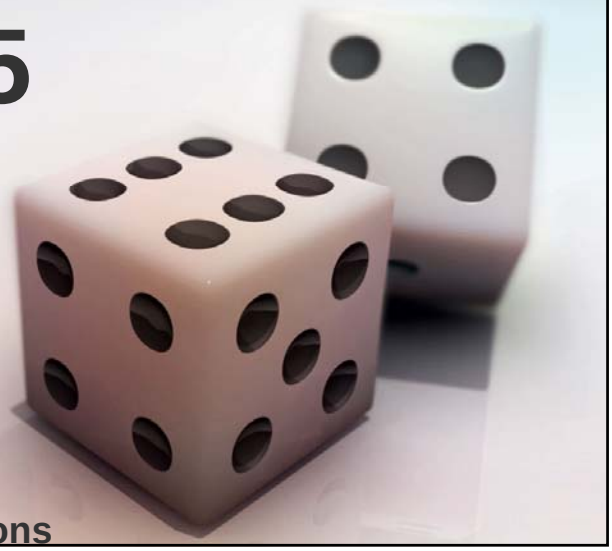
MA/CSSE 473 Day 15



Return Exam

Student questions

Towers of Hanoi

Subsets

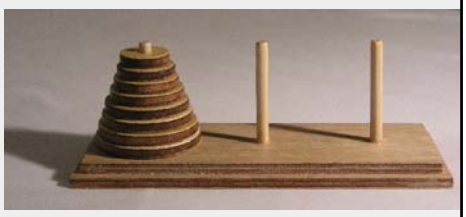
Ordered Permutations

MA/CSSE 473 Day 13

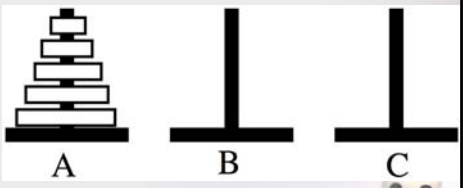
- Student Questions on exam or anything else
- Towers of Hanoi
- Subset generation – Gray code
- Permutations and order

Towers of Hanoi

- Move all disks from peg A to peg B
- One at a time
- Never place larger disk on top of a smaller disk



- Demo
- Code
- Recurrence and solution



Towers of Hanoi code

```
public class TowersOfHanoi {

   public TowersOfHanoi (int totalDisks)  {
      moveTower(totaldisks, 1, 2, 3);
   }

   /**************************************************************
     Moves the specified number of disks from one tower to another
     by moving a subtower of n-1 disks out of the way, moving one
     disk, then moving the subtower back. Base case of 1 disk.
   **************************************************************/
   private void moveTower (int numDisks, int start, int end, int temp) {
      if (numDisks == 1)
         moveOneDisk (start, end);
      else {
         moveTower (numDisks-1, start, temp, end);
         moveOneDisk (start, end);
         moveTower (numDisks-1, temp, end, start);
      }
   }

   /**************************************************************
     Prints a move from the start tower to the end tower.
   **************************************************************/
   private void moveOneDisk (int start, int end)   {
      System.out.println ("Move one disk from " + start + " to " + end);
   }
}
```

Recurrence for number of moves, and its solution?

Permutations and order

number	permutation	number	permutation
0	0123	12	2013
1	0132	13	2031
2	0213	14	2103
3	0231	15	2130
4	0312	16	2301
5	0321	17	2310
6	1023	18	3012
7	1032	19	3021
8	1203	20	3102
9	1230	21	3120
10	1302	22	3201
11	1320	23	3210

- Given a permutation of 0, 1, …, n-1, can we directly find the next permutation in the lexicographic sequence?
- Given a permutation of 0..n-1, can we determine its permutation sequence number?
- Given n and i, can we directly generate the i^{th} permutation of 0, …, n-1?

Subset generation

- **Goal:** generate all subsets of {0, 1, 2, …, N-1}
- **Bottom-up** (decrease-by-one) approach
- First generate S_{n-1}, the collection of all subsets of {0, …, N-2}
- Then $S_n = S_{n-1} \cup \{ S_{n-1} \cup \{n-1\} : s \in S_{n-1}\}$

Subset generation

- **Numeric approach:** Each subset of {0, …, N-1} corresponds to an bit string of length N where the i^{th} bit is 1 iff i is in the subset.
- So each subset can be represented by N bits.
- A simple loop generates them all in "numeric" order.

Subset generation

- **Minimal change algorithm**:
- flip exactly one bit each time we generate the next subset.
- Most common minimal-change approach: Binary-reflected Gray code.
  See the links in the announcements page and the schedule page.
- Transition sequences: which bit to flip
- 0
  010
  0102010
  010201030102010

Recap: Permutations and Order

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- Given a permutation of 0, 1, …, n-1, can we directly find the next permutation in the lexicographic sequence?
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Discovery time (with two partners)

- Which permutation follows each of these in lexicographic order?
  - 183647520 471638520
  - Try to write an algorithm for generating the next permutation, with only the current permutation as input.
- If the lexicographic permutations of the numbers [0, 1, 2, 3, 4, 5] are numbered starting with 0, what is the number of the permutation 14032?
  - General form? How to calculate efficiency?
- In the lexicographic ordering of permutations of [0, 1, 2, 3, 4, 5], which permutation is number 541?
  - How to calculate efficiently?