

**Due 10 minutes after class starts (9:10, 10:05, 11:00)**

1. (4) Show how to use the extended Euclid algorithm to find  $5^{-1} \pmod{29}$ . You will receive 1 point for the correct answer, and the other 4 for using extended Euclid correctly to get it.

$$29 = 5 \cdot 5 + 4$$

$$5 = 1 \cdot 4 + 1. \quad \text{GO } \gcd(29, 5) = 1. \quad \text{Thus 5 has an inverse (mod 29)}$$

Now work backwards:

$$1 = 5 - 4 = 5 - (29 - 5 \cdot 5) = 6 \cdot 5 - 29. \quad \text{Summary: } 1 = 6 \cdot 5 - 29.$$

**Thus 6 is the inverse of 5 (mod 29).**

**Check it: Not required.  $6 \cdot 5 = 30$ , which is one more than 29.**

2. (4) A *permutation* of a set **S** is a function from **S** to **S** that is **one-to-one and onto**.
3. (4) Fermat's little theorem says that if  $p$  is prime, then for all numbers  $a$  in  $\{1, 2, 3, \dots, p-1\}$   
 **$a^{p-1} \equiv 1 \pmod{p}$ . Equivalently,  $a^p \equiv a \pmod{p}$ .** Either answer counts as correct.
4. (4) Circle the correct answer. Fermat's little theorem can be used to show that a number **N**
- (a) is definitely prime
  - (b) is definitely not prime**
  - (c) whichever of those is true for  $N$
  - (d) neither

5. (4) What is a Fermat liar? **A number  $a$  such that  $a^{N-1} \equiv 1 \pmod{N}$  even though  $N$  is composite.**