CSSE 304 Exam #1 March 31, 2009 Name__________________________ Section 01__ 02__

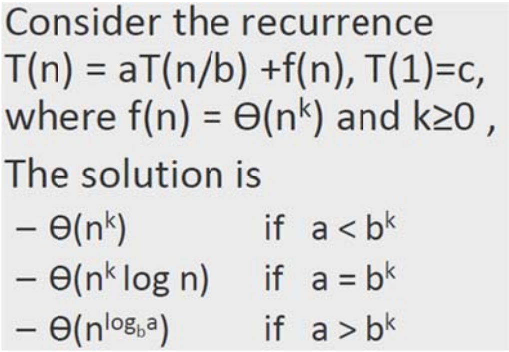
Resources allowed: Textbook, calculator, one 8.5 x 11 sheet of paper.
Resources not allowed: Anything that can communicate or has headphones/earphones.

Time is likely to be a factor on this exam; you should first do the problems you are confident about, and then do the rest.

Caution! It is possible to get so caught up in getting all of the points for one problem and spend so much time on it that you do not get to the other problems. Don't do that! I will be generous with partial credit if you have the main ideas.

For Instructor use:

Problem	Possible	Earned
1	7	
2	15	
3	12	
4	20	
5	15	
6	15	
7	6	
8	5	
9	5	
Total	100	



Consider the recurrence
$T(n) = aT(n/b) + f(n)$, $T(1)=c$,
where $f(n) = \Theta(n^k)$ and $k \geq 0$,
The solution is
- $\Theta(n^k)$ if $a < b^k$
- $\Theta(n^k \log n)$ if $a = b^k$
- $\Theta(n^{\log_b a})$ if $a > b^k$

MA/CSSE 473 Exam 1 Fall, 2010

1. (7) Show how to use the extended Euclid's algorithm to find the inverse of 3 (mod 40).

2. (15) I chose very small numbers for this problem, so the arithmetic can be done with minimal use of calculator. Because the possible answers come from a very small set, and could thus be gotten by trial and error, I insist that you show me how you get your answer, in a way that shows that you understand RSA.

   Bob has advertised his RSA public key as (N=55, e=27).
   Eve intercepts an encoded message meant for Bob: 4.

   What was the original message? ______

   Briefly show how you got your answer:

3. (12) T/F/IDK. Below you will find several statements. A statement is true (T) if it is always true. It is false (F) if there is at least one counterexample (sometimes false). You may also choose IDK to indicate that you do not know the answer. Point values: Correct answer: 3, incorrect answer: -1, IDK: 1, blank: 0.
   Circle one answer for each part.

   a. T F IDK If $f(N) \in O(N^2)$, $g(N) \in O(N^2)$, and $h(N) = f(N) / g(N)$, then $h(N) \in O(1)$.

   b. T F IDK If $f(N)) \in O(g(N))$ and $f(N) \in \Theta(g(N))$, then $f(N)) \in \Omega(g(N))$,

   c. T F IDK Worst-case running time for quicksort is asymptotically better than worst-case running time for merge sort.

   d. T F IDK Inorder and postorder traversals of a binary tree are sufficient to uniquely determine the tree.

4. (20) Suppose we have a 64-bit machine. Then we can multiply two 32-bit unsigned integers without overflow. Now suppose that we want to multiply two 512-bit positive integers, using one of the divide-and-conquer techniques from class. In the in-class algorithms, the recursion stops when the number of bits gets down to one. In this case we stop the recursion (and just do machine integer multiplication) when the number of bits gets down to 32.

    a. (6) If we use the standard approach to multiplication, how many 32-bit multiplications are done altogether?

    b. (6) If we use the algorithm based on Gauss's multiplication formula, how many 32-bit multiplications will be done?

    c. (4) If we only consider the costs of additions and multiplications, and if a 32-bit integer multiplication takes 5 times as long as a 32-bit integer addition, is the method from part (b) likely to be faster than the method from part (a) ? Show a calculation that leads to your conclusion.

    d. (4) Fill in the blank with the correct number:
    If we only consider the costs of additions and multiplications, and if a 32-bit integer multiplication takes

        ________ times as long as a 32-bit integer addition, then we can expect that the two approaches will take about the same amount of time. Show how you get your answer.

5. (15) Recall the "door in a wall" problem:

> You are facing a wall that stretches infinitely in both directions. There is a door in the wall, but you know neither how far away nor in which direction. You can see the door only when you are right next to it. Design an algorithm that enables you to reach the door by walking at most $O(n)$ steps where n is the (unknown to you) number of steps between your initial position and the door.

Suppose that you use this sub-optimal algorithm: Walk 1 step in one direction, then 4 steps in the other direction, then 9, then 16, then 25, 36, … So at each stage, you walk k^2 steps for some k, then turn around and walk $(k+1)^2$ steps in the other direction if you have not found the door.

(a) (5) After the stage in which you walk k^2 steps in one direction, how far are you from the original starting point? (for example, when k=3, you are 6 steps away from the start; what is the general formula?)

(b) (10) As a function of N (the distance from the original starting point to the door), what is the worst-case total number of steps taken before finding the door? Give a big-theta estimate and show how you get it.

6. (15) A *binary tree* is defined to be either (a) empty, or (b) a root node and two subtrees, T_L and T_R, each of which is a binary tree. The *height* of a binary tree is the length of the longest path from the root node to another node in the tree. Note that a tree with one node has height 0, and the empty tree has height -1. We can use the notation N(T) = the number of nodes in tree T, and H(T) = the height of tree T. Use mathematical induction and the given definition of binary tree to carefully prove that for every binary tree, $N(T) \leq 2^{H(T)+1} - 1$. . Be sure that your description makes it clear where and how you apply the induction hypothesis.

7. (6) This problem is based on the CACM interview with Donald Knuth that you were assigned to read.

   (a) Based on something in that article, briefly argue that Knuth would approve of my assigning you to implement the convex hull algorithms.

   (b) Based on something in that article, briefly argue that Knuth would not approve of my assigning you to implement the convex hull algorithms.

8. (5) If we use brute force integer multiplication and addition, and brute-force matrix multiplication, what is the big-theta running time (in terms of n) for multiplying n n×n matrices of n-bit integers? Circle one

   $\Theta(n^2)$ $\Theta(n^3)$ $\Theta(n^4)$ $\Theta(n^5)$ $\Theta(n^6)$ $\Theta(n^7)$ $\Theta(n^8)$ $\Theta(n^9)$ $\Theta(n^{10})$ $\Theta(n^{11})$ $\Theta(n^{12})$

9. (5) Find the big-theta solution of $T(N) = 9T(N/3) + 4N^2$

   Answer: $T(N) \in \Theta(\qquad)$