MA/CSSE 473 Day 19

Sleepsort
Finish Nim
Josephus problem
Transform and conquer examples

MA/CSSE 473 Day 19

- Announcement of Exam 2 allowed resource change
- **Student Questions**
- SleepSort demo
- Proofs of Nim strategy lemmas
- Josephus problem
- Transform and conquer – what's it all about?
- Instance simplification: presorting
- Instance simplification: Gaussian elimination and LU decomposition
- (Representation change: AVL trees)

Recap Multi-Pile Nim

- There are multiple piles of chips. Two players take turns by removing from any single pile at least one and at most all of that pile's chips. (The number of chips taken can vary from move to move)
- The winner is the player who takes the last chip.
- What is the winning strategy for 2-pile Nim?
- For the general case, consider the "Nim sum", $x \oplus y$, which is the integer obtained by bitwise XOR of corresponding bits of two non-negative integers x and y.
- What is $6 \oplus 3$?

Recap: Multi-Pile Nim Strategy

- Solution by C.L. Bouton:
- The first player has a winning strategy iff the nim sum of the "pile counts" is not zero.
- **Let's prove it.** Note that $\oplus$ is commutative and associative.
- Also note that for any non-negative integer k, $k \oplus k$ is zero.

Multi-Pile Nim Proof

- **Notation:**
  - Let $x_1, \ldots, x_n$ be the sizes of the piles before a move, and $y_1, \ldots, y_n$ be the sizes of the piles after that move.
  - Let $s = x_1 \oplus \ldots \oplus x_n$, and $t = y_1 \oplus \ldots \oplus y_n$.
- **Observe:** If the chips were removed from pile k, then $x_i = y_i$ for all $i \neq k$, and $x_k > y_k$.
- **Lemma 1:** $t = s \oplus x_k \oplus y_k$.
- **Lemma 2:** If $s = 0$, then $t \neq 0$.
- **Lemma 3:** If $s \neq 0$, it is possible to make a move such that t=0. [after proof, do an example].
- Proof of the strategy is then a simple induction. (It's a HW problem)
- **Example:** 3 piles, containing 7, 13, and 8 chips.

Josephus problem - background

- Flavius Josephus was a Jewish general and historian who lived and wrote in the 1st century AD
- Much of what we know about 1st century life in Israel (and the beginnings of Christianity) before and after the Roman destruction of the Jewish temple in 70 AD comes from his writings
- The "Josephus problem" is based on an odd suicide pact that he describes
  - He and his men stood in a circle and counted off
  - Every other person (or every third person, accounts vary) was killed
  - The last person was supposed to kill himself
  - He must have been the next-to-last person!
  - When it got down to two people, he persuaded the other person that they should surrender instead
- http://en.wikipedia.org/wiki/Josephus

Josephus Problem

- n people, numbered 1-n, are in a circle
- Count starts with 1
- Every 2nd person is eliminated
- The last person left, J(n), is the winner
- Examples: n=8, n=7
- J(1) = 1
- Solution if n is even:
- Solution if n is odd:
- Use it to find J(2) … J(8)
- Clever solution: cyclic bit shift left