MA/CSSE 473 Day 15

Main ideas from today:

1. Towers of Hanoi. Problem and the solution code are on the slide.

2. Recurrence relation for number of ToH moves:

   Examples: N=1
   N=2
   N=3
   N=4

   Closed form solution:

3. Is this algorithm decrease by a constant amount, decrease by a constant factor, or decrease by a variable amount?

4. Generate the power set (set of all subsets) of a set {0, 1, …, N-1}

5. **Bottom-up (decrease-by-one) approach**
   a. Generate S_{n-1}, the collection of the 2^{n-1} subsets of {0, 1, …, N-2}
   b. Then $S_n = S_{n-1} \cup \{ S_{n-1} \cup \{N-1\} : s \in S_{n-1}\}$

6. **Simple numeric approach:** Each subset of {0, 1, …, N-1} corresponds to an bit string of length N, where the i^{th} bit is 1 iff i is in the subset.

7. **Minimal change algorithm:** Binary-reflected Gray Code:
   flip one bit when moving from one subset to the next one.

   Transition sequence tells which bit to change as we move from one subset to the next.

8. Which permutation follows each of these in lexicographic order?

   183647520   471638520

9. Write an algorithm for generating the next permutation, given only N and the current permutation as input. You can express it in English if you wish.

10. If the lexicographic permutations of the numbers [0, 1, 2, 3, 4] are numbered starting with 0, what is the lexicographic number of the permutation 14032? How do you get this?

11. In the lexicographic ordering of permutations of [0, 1, 2, 3, 4, 5], which permutation is number 541?

12. Write an algorithm which, given a permutation of the numbers 0, 1, …, N-1, calculates its (zero-based) position in the lexicographic ordering of all of the permutations of 0..N-1.

13. (If you have extra time) Polynomial evaluation (do it on the back of this page):
    Given a polynomial $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$
    a. How can we efficiently evaluate p(c) for some number c?
    b. Apply this to evaluation of "31427894" or any other string that represents a positive integer.
    c. Write and analyze (pseudo)code.