HW 01 textbook problems and hints

This document is intended to supplement, not replace, the actual assignment document. Read that one first. If there are discrepancies in point values or problems between this document and the assignment document, assume that the assignment document is correct, and tell me so I can fix this one. **This notice applies to all of my "problems and hints" documents.**

#1 **Section 1.1 (10 points)** Problem 12 [11 from 2nd edition]

> 11. *Locker doors* There are n lockers in a hallway numbered sequentially from 1 to n. Initially, all the locker doors are closed. You make n passes by the lockers, each time starting with locker #1. On the ith pass, $i = 1, 2, ..., n$, you toggle the door of every ith locker: if the door is closed, you open it, if it is open, you close it. For example, after the first pass every door is open; on the second pass you only toggle the even-numbered lockers (#2, #4, ...) so that after the second pass the even doors are closed and the odd ones are opened; the third time through you close the door of locker #3 (opened from the first pass), open the door of locker #6 (closed from the second pass), and so on. After the last pass, which locker doors are open and which are closed? How many of them are open?

Author's Hint

> 11. Tracing the algorithm by hand for, say, $n = 10$ and studying its outcome should help answering both questions.

#2 **1.2 (5)** (four people and a flashlight)

> 2. *New World puzzle* There are four people who want to cross a bridge; they all begin on the same side. You have 17 minutes to get them all across to the other side. It is night, and they have one flashlight. A maximum of two people can cross the bridge at one time. Any party that crosses, either one or two people, must have the flashlight with them. The flashlight must be walked back and forth; it cannot be thrown, for example. Person 1 takes 1 minute to cross the bridge, person 2 takes 2 minutes, person 3 takes 5 minutes, and person 4 takes 10 minutes. A pair must walk together at the rate of the slower person's pace. For example, if person 1 and person 4 walk across first, 10 minutes have elapsed when they get to the other side of the bridge. If person 4 returns the flashlight, a total of 20 minutes have passed and you have failed the mission. (Note: According to a rumor on the Internet, interviewers at a well-known software company located near Seattle have given this problem to interviewees.)

Author's Hint

> 2. Unlike the Old World puzzle of Problem 1, the first move solving this puzzle is not obvious.

#3 **1.3 (5)** (are n given points on circumference of the same circle?). Input: a list of coordinates, output: boolean.
You can be brief, but do not be so vague that I cannot tell whether you really know how to do this.

9. Design an algorithm for the following problem: Given a set of n points in the Cartesian plane, determine whether all of them lie on the same circumference.

Author's Hint

9. Assume that the circumference in question exists and find its center first. Also, do not forget to give a special answer for $n \leq 2$.

Instructor's Clarification

Input: a list of coordinates
Output: boolean.
Your description can be brief, but not so vague that I cannot tell whether you really know how to do this.

#4, 5, 6 **1.4 (6, 5, 6)** For #4, See pages Levitin 28-30 for definitions and representations

4. a. Let A be the adjacency matrix of an undirected graph. Explain what property of the matrix indicates that
   i. the graph is complete.
   ii. the graph has a loop, i.e., an edge connecting a vertex to itself.
   iii. the graph has an isolated vertex, i.e., a vertex with no edges incident to it.

   b. Answer the same questions for the adjacency list representation.

Author's Hint

4. Just use the definitions of the graph properties in question and data structures involved.

5. Give a detailed description of an algorithm for transforming a free tree into a tree rooted at a given vertex of the free tree.

Instructor note on problem 5 (1.4.4):) The algorithm is given a reference to a node of the free tree. Assume that the free tree is represented as a graph with adjacency lists, and that each node of the rooted tree has a list of the nodes that are adjacent to it. The algorithm should return a reference to the root node of the rooted tree that it constructs.
The assignment document contains (at the very end of the document) an example.

Author's Hint

5. There are two well-known algorithms that can solve this problem. The first uses a stack, the second uses a queue. Although these algorithms are discussed later in the book, do not miss this chance to discover them by yourself!

10. *Anagram checking* Design an algorithm for checking whether two given words are anagrams, i.e., whether one word can be obtained by permuting the letters of the other. (For example, the words *tea* and *eat* are anagrams.)

10. There are several algorithms for this problem. Keep in mind that the words may contain multiple occurrences of the same letter.

Instructor's Note: Note that it says "anagram", not "palindrome".

#7 Not from the textbook (15)

7.   (5) (combinatorial practice, thanks to former RHIT faculty member Salman Azhar) .
This problem is partly an exercise in careful reading. Not an easy problem. Each element of nonempty set A is a set of four distinct random digits; each digit is in range 1..9 (inclusive). Example: A = {{2, 4, 7, 5}, {1, 4, 8, 3}, {1, 3, 5, 9}}. Construct set B from A as follows. For each element of A (that element is a set of four single-digit numbers), construct the corresponding element of B as the sum of the 24 four-digit integers that are all of the possible permutations of that element of A.   So if A is a set of k sets, B is a set of k numbers. Find (and show how you get it) the smallest value of the GCD (greatest common divisor) of all of the elements of all possible instances of B. Note that the GCD of the elements in a particular set B depends on which sets are in the particular A from which B is derived. So we are looking for

min({gcd(B) : B is the set of the sums of all permutations derived from each of 4-element sets in A}),
where the minimum is taken over *all* A's which are nonempty sets of 4-element sets as described above.

Note: If the answer to this problem is a number S, there are two things to show:
1. S is a factor of every GCD that cones from every set A.
   A brief mathematical argument is needed for this part.
2. There actually is a specific set A that leads to S as the GCD. Show me such an A.

#8 Not from the textbook (10)

8.   (10) Read through the review of Mathematical induction that is linked from the
Resources column of Session 2 on the Schedule Page.

This is an enhanced composite of some materials that I used when we used to emphasize induction more in CSSE 230. Even if you already feel quite comfortable with both ordinary and strong induction, be sure to read the part on how not to do induction.
Other sources on induction: Weiss 7.2, Grimaldi Chapter 4.

Let the Fibonacci numbers be as defined on page 80 [78] of Levitin.   On pages 80-82 [79-80], the author derives an explicit formula (2.9)[(2.11)] for the nth Fibonacci number. Another approach

(the one Weiss uses) is to produce the formula "by magic" and use mathematical induction to prove it. That "proof by induction" approach is what you are to do for this problem. Be sure that in your induction proof you make it explicit what the induction hypothesis is and how it is used.

Hint: you might first want to prove (induction is not needed for this part) the simple relationship among 1, ϕ, and ϕ^2 , namely $1+ \phi = \phi^2$, which implies the same relationship among ϕ^k, ϕ^{k+1}, and ϕ^{k+2}. Note that ϕ and $\hat\phi$ are defined on page 80 [80] of Levitin.

Note that this is essentially the same as Weiss problem 7.8, which I always assign when I teach CSSE 230.

#9 Not from the textbook (5)

9.　　　　　(5) Fake 8. There are 8 identical-looking coin, but one of them is lighter than the others. Our only tool is a pan balance, which will tell us whether the coins in the two pans have equal weight, or whether the left or right side is heavier. What is the minimum number of weighings required to guarantee that we know which is the fake coin? Briefly explain your algorithm. Hint: A balance pan can hold more than one coin at a time.