MA/CSSE 473 – Design and Analysis of Algorithms

Homework 6 (103 points total) Updated for Summer, 2016

This assignment is slightly longer than the typical 473 assignment, so I am splitting it into two parts, due on different days.

When a problem is given by number, it is from the textbook. 1.1.2 means "problem 2 from section 1.1" .

Problems for enlightenment/practice/review (not to turn in, but you should think about them):

How many of them you need to do serious work on depends on you and your background. I do not want to make everyone do one of them for the sake of the (possibly) few who need it. You can hopefully figure out which ones you need to do.

3.5.2 [5.2.2] (adjacency matrix *vs* adjacency list for DFS)
3.5.7 [5.2.7] (Use BFS/DFS to find a graph's connected components)
3.5.10 [5.2.10] (DFS and mazes)
4.1.8 [5.1.5] (insertion sort sentinel)
5.1.125.1.10 (Shell's sort) This should be review from 230
4.2.1 [5.3.1] (Topological sort examples)
4.2.2 [5.3.2] (Theoretical properties of topological sort)
4.3.1 [5.4.1] (Reasonableness of generating all permutations, subsets of a 25-element set)
4.3.9 [5.4.9] (Generation of binary reflected Gray Code based on bit-flipping)

Problems to write up and turn in:

1. (6) 3.5.3 [5.2.3] (independence of properties from specific DFS traversals) Explain your answers.
2. (10) 3.5.8a [5.2.8a] (Bipartite graph checking using DFS)
3. (5) 4.1.1 [5.1.1] (Ferrying Soldiers)
4. (5) 4.1.4 [5.1.3] (generate power set)
5. (5) (not in book) [5.1.9] (binary insertion sort efficiency).

   Binary insertion sort uses binary search to find an appropriate position to insert A[i] among the previously sorted A[0] ≤ … ≤ A[I - 1]. Determine the worst-case efficiency class of this algorithm. I.e. get big-Θ time for number of comparisons and number of moves.

6. (9) 4.2.6 [5.3.6] (finding dag sources) Be sure to do all three parts.
7. (9) 4.2.9 [5.3.9] (Strongly connected components of a digraph)
8. (15) (Miller-Rabin test) For this problem you will need the excerpt from the Dasgupta book that is posted on Moodle, and/or Weiss section 9.6.
   Let N = 1729 (happens to be a Carmichael number, but you should not assume that as you discover the answers) for all parts of this problem.
   (a) How many values of **a** in the range 1..1728 pass the Fermat test [i.e. $a^{1728} \equiv 1 \pmod{1729}$]?
   (b) For how many of these "Fermat test positive" values of **a** from part (a) does the Miller-Rabin test provide a witness that N is actually composite?
   (c) If we pick **a** at random from among 1, 2,..,1728, what is the probability that running the Miller-Rabin test on **a** will show that N is composite? I.e., Rabin showed that for any N, the probability is at least 75% for every N; what is that probability for the N=1729 case?

   [**Hint**: writing some code is likely (probably necessary) to help you solve this problem. If you do that, include the code in your submission].

9. (9) 4.3.2 [5.4.2] (Examples of permutation generation algorithms)

   You do not have to write any code, but you can do it that way if you wish.

10. (10) 4.3.10 [5.4.10] (Generation of all k-combinations from an n-element set)
11. (10) 4.3.11 [5.4.11] (Generation of binary reflected Gray code based on Tower of Hanoi moves.
12. (10) 4.3.12 [not in 2nd edition] Fair attraction. See the "problems" document for details.