MA/CSSE 473 Exam #2 Fall, 2012 Name_____ Section 01__ 02__

Resources allowed: Calculator, two 8.5 x 11 sheet of paper.
Resources not allowed: Textbook, computer, anything that can communicate or has headphones/earphones.

It is possible to get so caught up in getting all of the points for one problem and spend so much time on it that you do not get to the other problems. Don't do that! I will be generous with partial credit if you have the main ideas. You should first do the problems you are confident about, and then do the rest.

IN PARTICULAR: The first problem may be the most involved. I put it first so you would know what you are facing. Don't spend so much time on it that you do not get to some easier problems.

For Instructor use:

Problem	Possible	Earned
1	20	
2	10	
3	5	
4	15	
5	10	
6	10	
7	10	
8	20	
Total	100	

Consider the recurrence
$T(n) = aT(n/b) + f(n)$, $T(1)=c$,
where $f(n) = \Theta(n^k)$ and $k \geq 0$,
The solution is

- $\Theta(n^k)$ if $a < b^k$
- $\Theta(n^k \log n)$ if $a = b^k$
- $\Theta(n^{\log_b a})$ if $a > b^k$

1. (20) Suppose we start with an empty AVL tree, and insert the numbers 1, 2, 3, …, 2^k-1, into that tree, one at a time, in the order given. (You may want to do some examples so you can see a pattern.
    a. (10) What is the height of the resulting tree?

    b. (5) How many rotations are done altogether (express your answer as a function of k)?

    c. (5)We saw the formula from part b in the analysis of another algorithm. Which algorithm is it?

2. (16) The N elements of a binary (min) heap are stored in positions 1-6 of an array. If the original arrangement of the array elements is not a heap, we examined two algorithms for making it into a heap. (1) top down, repeated insert of elements 2-N to build up the heap, and (2) bottom-up heap construction, using the buildHeap (also known as *fixheap*) algorithm. If the original array is as shown below, show the results of running the two different algorithms. Be very careful because it will be difficult to give partial credit. If it helps you get your answers, feel free to draw the binary trees represented by the arrays.

Original array		4	3	6	5	2	1
	0	1	2	3	4	5	6
Heap after repeated insert on original array							
Heap after buildheap on original array							

3. (5) What is the theoretical minimum asymptotic running time for generating a list of all permutations of the first n integers? Is there a simple algorithm that achieves this minimum?

4. (15) It is easy to show that 45 is a composite number. Bur so you can demonstrate your knowledge of primality testing without having to deal with big numbers, I am using that small number here. Below you will find a table of powers of various numbers, mod 45. Each row contains various powers of one number.

   ____ is a value of *a* that demonstrates that 45 is composite by failing the Fermat test

   __−(mod 45) is the power of a number in the table that shows that this *a* fails the test.

   ____ is a value of *a* that passes the Fermat test, but that demonstrates that 45 is composite by the Rabin-Miller test.

   The sequence of powers of this *a* that the Rabin-Miller test uses is _____

   This sequence shows that 45 is composite because it contains a _____

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
2	4	8	16	32	19	38	31	17	34	23	1	2	4	8	16	32	19	38	31	17	34	23	1	2	4	8	16	32	19	38	31	17	34	23	1	2	4	8	16	32	19	38	31	17
3	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18
4	16	19	31	34	1	4	16	19	31	34	1	4	16	19	31	34	1	4	16	19	31	34	1	4	16	19	31	34	1	4	16	19	31	34	1	4	16	19	31	34	1	4	16	19
5	25	35	40	20	10	5	25	35	40	20	10	5	25	35	40	20	10	5	25	35	40	20	10	5	25	35	40	20	10	5	25	35	40	20	10	5	25	35	40	20	10	5	25	35
6	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36
7	4	28	16	22	19	43	31	37	34	13	1	7	4	28	16	22	19	43	31	37	34	13	1	7	4	28	16	22	19	43	31	37	34	13	1	7	4	28	16	22	19	43	31	37
8	19	17	1	8	19	17	1	8	19	17	1	8	19	17	1	8	19	17	1	8	19	17	1	8	19	17	1	8	19	17	1	8	19	17	1	8	19	17	1	8	19	17	1	8
9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
11	31	26	16	41	1	11	31	26	16	41	1	11	31	26	16	41	1	11	31	26	16	41	1	11	31	26	16	41	1	11	31	26	16	41	1	11	31	26	16	41	1	11	31	26
12	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27
13	34	37	31	43	19	22	16	28	4	7	1	13	34	37	31	43	19	22	16	28	4	7	1	13	34	37	31	43	19	22	16	28	4	7	1	13	34	37	31	43	19	22	16	28
14	16	44	31	29	1	14	16	44	31	29	1	14	16	44	31	29	1	14	16	44	31	29	1	14	16	44	31	29	1	14	16	44	31	29	1	14	16	44	31	29	1	14	16	44
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	31	1	16	31	1	16	31	1	16	31	1	16	31	1	16	31	1	16	31	1	16	31	1	16	31	1	16	31	1	16	31	1	16	31	1	16	31	1	16	31	1	16	31	1
17	19	8	1	17	19	8	1	17	19	8	1	17	19	8	1	17	19	8	1	17	19	8	1	17	19	8	1	17	19	8	1	17	19	8	1	17	19	8	1	17	19	8	1	17
18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18
19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19
20	40	35	25	5	10	20	40	35	25	5	10	20	40	35	25	5	10	20	40	35	25	5	10	20	40	35	25	5	10	20	40	35	25	5	10	20	40	35	25	5	10	20	40	35
21	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36
22	34	28	31	7	19	13	16	37	4	43	1	22	34	28	31	7	19	13	16	37	4	43	1	22	34	28	31	7	19	13	16	37	4	43	1	22	34	28	31	7	19	13	16	37
23	34	17	31	38	19	32	16	8	4	2	1	23	34	17	31	38	19	32	16	8	4	2	1	23	34	17	31	38	19	32	16	8	4	2	1	23	34	17	31	38	19	32	16	8
24	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9
25	40	10	25	40	10	25	40	10	25	40	10	25	40	10	25	40	10	25	40	10	25	40	10	25	40	10	25	40	10	25	40	10	25	40	10	25	40	10	25	40	10	25	40	10
26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26	1	26
27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27
28	19	37	1	28	19	37	1	28	19	37	1	28	19	37	1	28	19	37	1	28	19	37	1	28	19	37	1	28	19	37	1	28	19	37	1	28	19	37	1	28	19	37	1	28
29	31	44	16	14	1	29	31	44	16	14	1	29	31	44	16	14	1	29	31	44	16	14	1	29	31	44	16	14	1	29	31	44	16	14	1	29	31	44	16	14	1	29	31	44
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	16	1	31	16	1	31	16	1	31	16	1	31	16	1	31	16	1	31	16	1	31	16	1	31	16	1	31	16	1	31	16	1	31	16	1	31	16	1	31	16	1	31	16	1
32	34	8	31	2	19	23	16	17	4	38	1	32	34	8	31	2	19	23	16	17	4	38	1	32	34	8	31	2	19	23	16	17	4	38	1	32	34	8	31	2	19	23	16	17
33	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18	9	27	36	18
34	31	19	16	4	1	34	31	19	16	4	1	34	31	19	16	4	1	34	31	19	16	4	1	34	31	19	16	4	1	34	31	19	16	4	1	34	31	19	16	4	1	34	31	19
35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35	10	35
36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36
37	19	28	1	37	19	28	1	37	19	28	1	37	19	28	1	37	19	28	1	37	19	28	1	37	19	28	1	37	19	28	1	37	19	28	1	37	19	28	1	37	19	28	1	37
38	4	17	16	23	19	2	31	8	34	32	1	38	4	17	16	23	19	2	31	8	34	32	1	38	4	17	16	23	19	2	31	8	34	32	1	38	4	17	16	23	19	2	31	8
39	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9	36	9
40	25	10	40	25	10	40	25	10	40	25	10	40	25	10	40	25	10	40	25	10	40	25	10	40	25	10	40	25	10	40	25	10	40	25	10	40	25	10	40	25	10	40	25	10
41	16	26	31	11	1	41	16	26	31	11	1	41	16	26	31	11	1	41	16	26	31	11	1	41	16	26	31	11	1	41	16	26	31	11	1	41	16	26	31	11	1	41	16	26
42	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27	9	18	36	27
43	4	37	16	13	19	7	31	28	34	22	1	43	4	37	16	13	19	7	31	28	34	22	1	43	4	37	16	13	19	7	31	28	34	22	1	43	4	37	16	13	19	7	31	28
44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44	1	44

5. (10) If we start our numbering at 0, what is the 371st lexicographic permutation of {0,1,2,3,4,5}? Show how you get it.

6. (10) Write an efficient algorithm for finding x^N, where x is a floating-point number and N is a non-negative integer.

7. (10) This is Levitin's problem 6.5.12. Write an algorithm that, given a set of n data points (x_i, y_i), where no two x_i are the same, finds a polynomial p(x) of degree at most n-1 such that $p(x_i) = y_i$ for all $i=1, 2,\ldots,n$. A high-level description is sufficient.

8. (20) Boyer-Moore problem. Suppose the pattern is "APPLE APP".

   (a) (12) Calculate the bad character table and the good suffix table for this pattern.

   (b) (8) Show the Boyer-Moore matching process when searching for this pattern in the following string:

BPPLE APPLEAAPPAPPAPPAPPLE PAPAPPLE APPLE APP