

5. The following lemma can be used to prove that Kruskal's algorithm produces a MST. Prove it.

Lemma: Let G be a weighted connected graph with a MST T ; let G' be any subgraph of T , and let C be any connected component of G' .

If we add to C an edge $e=(v,w)$ that has minimum-weight among all of the edges that have one vertex in C and the other vertex not in C , then G has an MST that contains the union of G' and e . [Let v be our name for the vertex of e that is in C , and w our name for the vertex of e that is not in C].

6. Use the above lemma to prove that Kruskal's algorithm is correct:

Claim: After every step of Kruskal's algorithm, we have a set of edges that is part of an MST of G

Proof of claim: Base case ...

Induction Assumption: before adding an edge we have a subgraph of an MST

We must show that after adding the next edge we have a subgraph of an MST

Details:

7. Data Structures for Prim's algorithm.