

7. Prove: Let $S = \{1, 2, \dots, p-1\}$. For all a in S :

Lemma: Multiplying all of the numbers in S by $a \pmod{p}$ permutes S . I.e. $\{a \cdot n \pmod{p} : n \in S\} = S$

8. Use the lemma to prove Fermat's little theorem.

9. Note that the inverse of Fermat's little theorem is not true!

10. **Prove:** If a is a number that is relatively prime to N such that a^{N-1} is not congruent to $1 \pmod{N}$, then that same condition must be true for at least half of the numbers in the range $1 \dots N-1$.