HW 01 textbook problems and hints

Section 1.1 (problem 1 – 5 points) Problem 12 [11 from 2nd edition]

11. *Locker doors* There are n lockers in a hallway numbered sequentially from 1 to n. Initially, all the locker doors are closed. You make n passes by the lockers, each time starting with locker #1. On the ith pass, $i = 1, 2, ..., n$, you toggle the door of every ith locker: if the door is closed, you open it, if it is open, you close it. For example, after the first pass every door is open; on the second pass you only toggle the even-numbered lockers (#2, #4, ...) so that after the second pass the even doors are closed and the odd ones are opened; the third time through you close the door of locker #3 (opened from the first pass), open the door of locker #6 (closed from the second pass), and so on. After the last pass, which locker doors are open and which are closed? How many of them are open?

Author's Hint

11. Tracing the algorithm by hand for, say, $n = 10$ and studying its outcome should help answering both questions.

1.2 (2 - 5) (four people and a flashlight)

2. *New World puzzle* There are four people who want to cross a bridge; they all begin on the same side. You have 17 minutes to get them all across to the other side. It is night, and they have one flashlight. A maximum of two people can cross the bridge at one time. Any party that crosses, either one or two people, must have the flashlight with them. The flashlight must be walked back and forth; it cannot be thrown, for example. Person 1 takes 1 minute to cross the bridge, person 2 takes 2 minutes, person 3 takes 5 minutes, and person 4 takes 10 minutes. A pair must walk together at the rate of the slower person's pace. For example, if person 1 and person 4 walk across first, 10 minutes have elapsed when they get to the other side of the bridge. If person 4 returns the flashlight, a total of 20 minutes have passed and you have failed the mission. (Note: According to a rumor on the Internet, interviewers at a well-known software company located near Seattle have given this problem to interviewees.)

Author's Hint

2. Unlike the Old World puzzle of Problem 1, the first move solving this puzzle is not obvious.

1.3 (3 - 5) (are n given points on circumference of the same circle?). Input: a list of coordinates, output: boolean. You can be brief, but do not be so vague that I cannot tell whether you really know how to do this.

9. Design an algorithm for the following problem: Given a set of n points in the Cartesian plane, determine whether all of them lie on the same circumference.

Author's Hint

9. Assume that the circumference in question exists and find its center first. Also, do not forget to give a special answer for $n \leq 2$.

Instructor's Clarification

Input: a list of coordinates
Output: boolean.
Your description can be brief, but not so vague that I cannot tell whether you really know how to do this.

1.4 (4 – 6, 5 – 5, 6 - 3)

4. a. Let A be the adjacency matrix of an undirected graph. Explain what property of the matrix indicates that
   i. the graph is complete.
   ii. the graph has a loop, i.e., an edge connecting a vertex to itself.
   iii. the graph has an isolated vertex, i.e., a vertex with no edges incident to it.

   b. Answer the same questions for the adjacency list representation.

Author's Hint

4. Just use the definitions of the graph properties in question and data structures involved.

5. Give a detailed description of an algorithm for transforming a free tree into a tree rooted at a given vertex of the free tree.

Instructor note on problem 5 (1.4.4):) The algorithm is given a reference to a node of the free tree. Assume that the free tree is represented as a graph with adjacency lists, and that each node of the rooted tree has a list of the nodes that are adjacent to it. The algorithm should return a reference to the root node of the free tree that it constructs.

Author's Hint

5. There are two well-known algorithms that can solve this problem. The first uses a stack, the second uses a queue. Although these algorithms are discussed later in the book, do not miss this chance to discover them by yourself!

10. *Anagram checking* Design an algorithm for checking whether two given words are anagrams, i.e., whether one word can be obtained by permuting the letters of the other. (For example, the words *tea* and *eat* are anagrams.)

Author's Hint

10. There are several algorithms for this problem. Keep in mind that the words may contain multiple occurrences of the same letter.

Instructor's Note: Note that it says "anagram", not "palindrome".

Not from the textbook (7 – 5, 8 – 10)

7. (5) (combinatorial practice, thanks to former RHIT faculty member Salman Azhar) Each element of nonempty set A is a set of four distinct random digits, each digit in range 1..9 (inclusive) 1 and 9. Example: A = {{2, 4, 7, 5}, {1, 4, 8, 3}, {1, 3, 5, 9}}. Construct set B as follows. For each element of A, we construct a single element of B as the sum of the 24 four-digit numbers that are all possible permutations of that element of A. So if A is a set of k sets, B is a set of k numbers. What is the smallest possible value of the GCD (greatest common divisor) of all of the elements of B? Note that the GCD of the elements in B could possibly depend on which sets are in the particular A from which B is derived.

8. (10) Read through the review of Mathematical induction that is linked from the Resources column of Session 2 on the Schedule Page.

This is an enhanced composite of some materials that I used when we used to emphasize induction more in CSSE 230. Even if you already feel quite comfortable with both ordinary and strong induction, be sure to read the part on how not to do induction.
Other sources on induction: Weiss 7.2, Grimaldi Chapter 4.

Let the Fibonacci numbers be as defined on page 80 [78] of Levitin. On pages 80-82 [79-80], the author derives an explicit formula (2.9)[(2.11)] for the nth Fibonacci number. Another approach (the one Weiss uses) is to produce the formula "by magic" and use mathematical induction to prove it. That "proof by induction" approach is what you are to do for this problem. Be sure that in your induction proof you make it explicit what the induction hypothesis is and how it is used.

Hint: you might first want to prove (induction is not needed for this part) the simple relationship among 1, ϕ, and ϕ^2 , namely $1+ \phi = \phi^2$, which implies the same relationship among ϕ^k, ϕ^{k+1}, and ϕ^{k+2}. Note that ϕ and $\hat{\phi}$ are defined on page 80 [80] of Levitin.