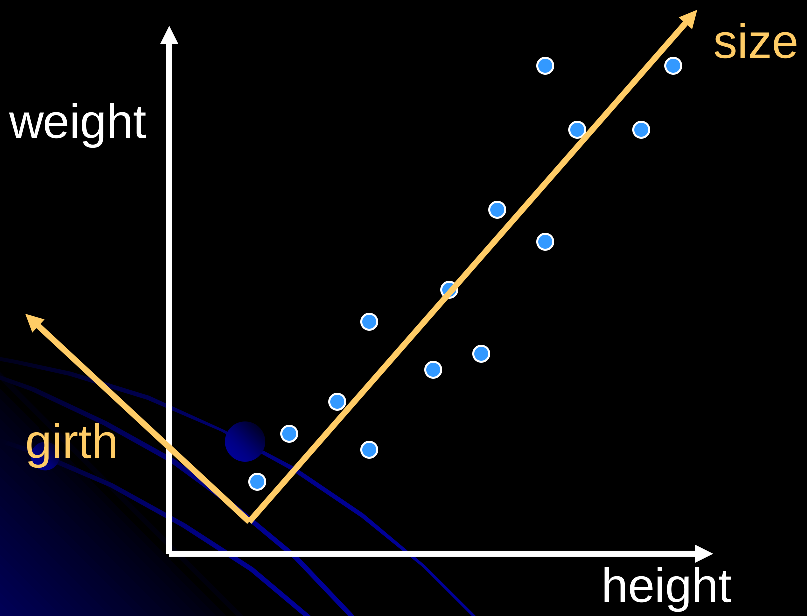
CSSE463: Image Recognition Day 10

- Lab 3 due Weds
- Today:
  - finish circularity
  - region *orientation*: principal axes
- Questions?

Principal Axes

- Gives orientation and elongation of a region
  - Demo

Some intuition from statistics

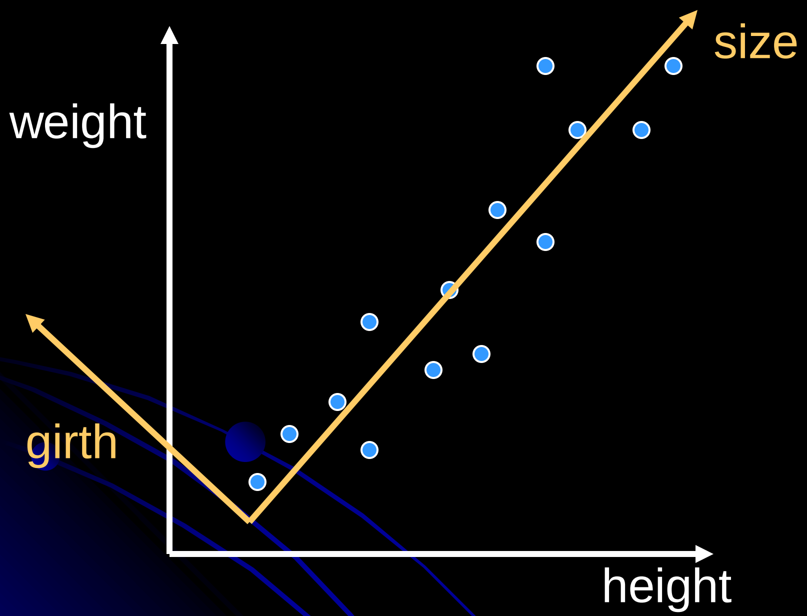


- Sometimes changing axes can give more intuitive results

The size axis is the principal component: the dimension giving greatest variability.

The girth axis is perpendicular to the size axis. It is uncorrelated and gives the direction of least variability.

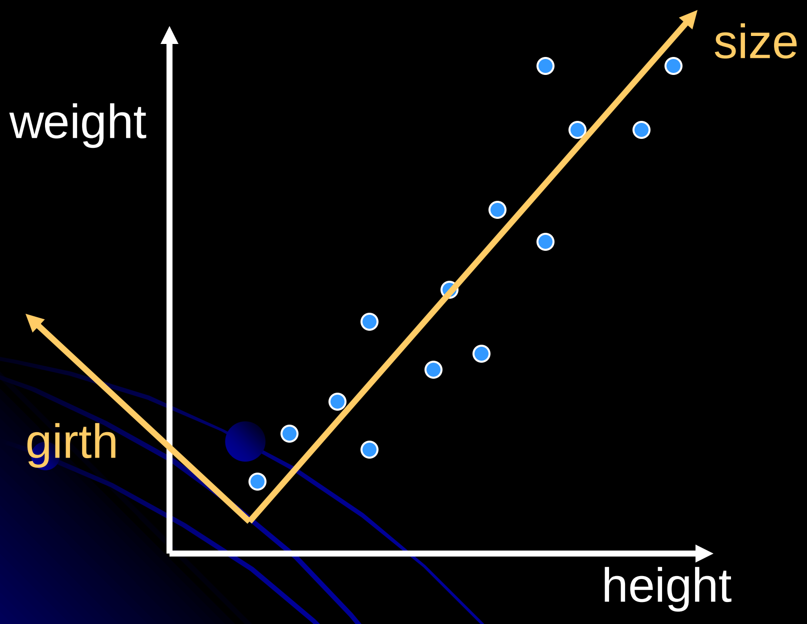
How to find principal components?



- How would you find these?

Answer this now on quiz

How to find principal components?



- Recall from statistics, for distributions of 2 variables, **variance** of each variable and also **covariance** between the 2 variables are defined.

$$\sigma_{xx} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x})$$

$$\sigma_{xy} = \sigma_{yx} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$\sigma_{yy} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})(y_i - \bar{y})$$

n=# of data (points in region)

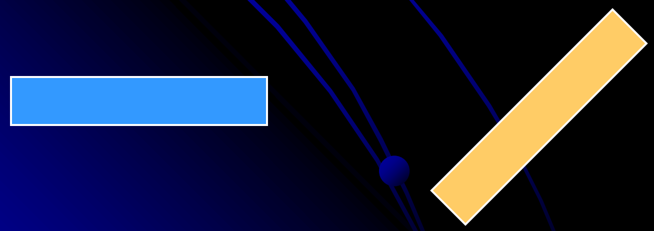
Intuitions

$$\sigma_{xx} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x})$$

$$\sigma_{xy} = \sigma_{yx} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$\sigma_{yy} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})(y_i - \bar{y})$$

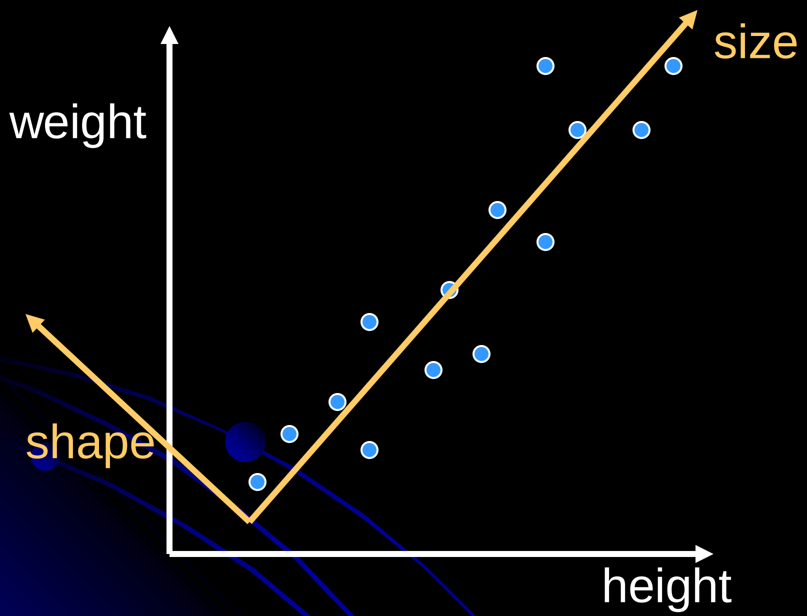
$$c = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$



- σ_{xx}: How much x alone varies
- σ_{xy}: How much x and y co-vary (are they correlated or independent?)
- σ_{yy}: How much y alone varies
- Together, they form the covariance matrix, C
- Examples on board

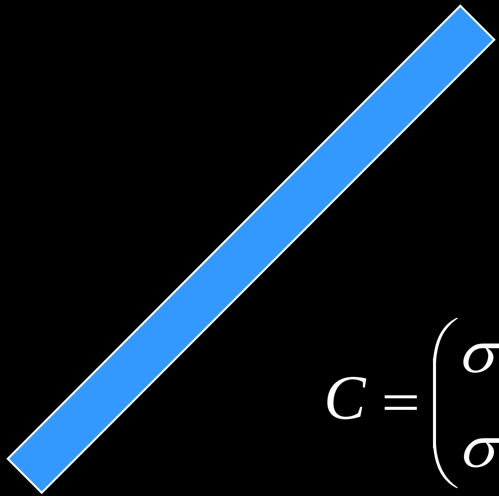
Q3,Q4

Theorem (w/o proof)



- The eigenvectors of the covariance matrix give the directions of variation, sorted from the one corresponding to the largest eigenvalue to the one corresponding to the smallest eigenvalue.
- Because the matrix is symmetric, the eigenvalues are guaranteed to be positive real numbers, and eigenvectors are orthogonal

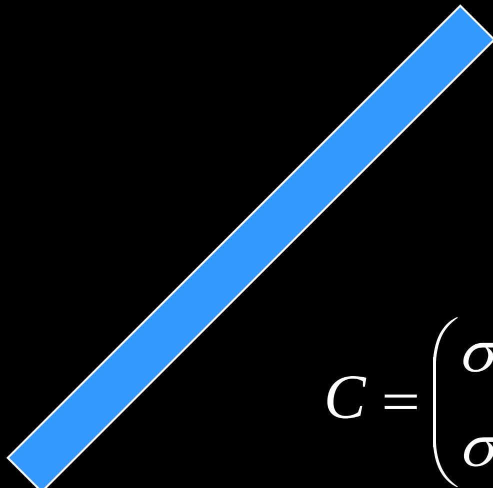
Application to images



$$C = \begin{pmatrix} \sigma_{cc} & \sigma_{cr} \\ \sigma_{rc} & \sigma_{rr} \end{pmatrix}$$

- Can find out the shape's **principal axis** and its **elongation**
- Consider the points in a region to be a 2D distribution of points.

Application to images



$$C = \begin{pmatrix} \sigma_{cc} & \sigma_{cr} \\ \sigma_{rc} & \sigma_{rr} \end{pmatrix}$$

- Consider the points in a region to be a 2D distribution of points.
  - 2 vectors: [r,c] (as returned by find)
  - Use covariance formulas
    - (but replace x with c and y with r)
- The elements of the covariance matrix are called second-order spatial moments
- **Different than the spatial color moments in the sunset paper!**

Q5-Q6

How to find principal axes?

1. Calculate spatial covariance matrix using previous formulas:
2. Find eigenvalues, λ_1, λ_2, and eigenvectors, v1, v2.
3. **Direction of principal axis** is direction of eigenvector corresponding to largest eigenvalue
4. Finally, a measure of the **elongation** of the shape is:

$$C = \begin{pmatrix} \sigma_{cc} & \sigma_{cr} \\ \sigma_{rc} & \sigma_{rr} \end{pmatrix}$$

$$elongation = \sqrt{\frac{\lambda_{\max}}{\lambda_{\min}}}$$

Lab 4

- Could you use the region properties we've studied to distinguish different shapes (squares, rectangles, circles, ellipses, triangles, …)?