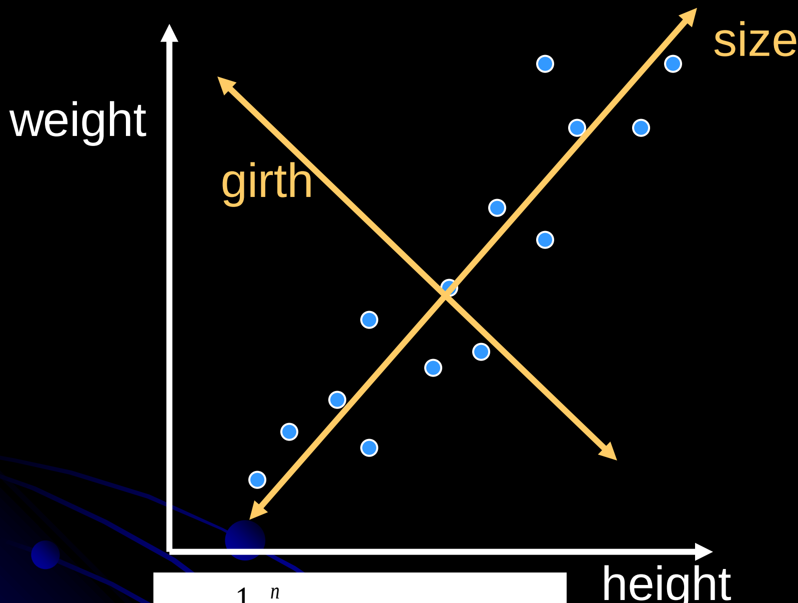
CSSE463: Image Recognition Day 26

- This week
  - Today: Applications of PCA
  - Weds night: k-means lab due.
  - Thursday: template matching for object recognition
  - Sunday night: project plans and prelim work due
- Questions?

Principal Components Analysis



$$\sigma_{xx} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x})$$

$$\sigma_{xy} = \sigma_{yx} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$\sigma_{yy} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})(y_i - \bar{y})$$

$$c = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

- Given a set of samples, find the direction(s) of greatest variance.
- We've done this!
- **Spatial moments**
  - Principal axes are eigenvectors of covariance matrix
  - Eigenvalues gave relative importance of each dimension
  - Note that each point can be represented in 2D using the new coordinate system defined by the eigenvectors
  - The 1D representation obtained by projecting the point onto the principal axis is a reasonably-good approximation

Covariance Matrix (using matrix operations)

Place the points in their own column.

$$F = \begin{pmatrix} x_1 & x_2 & x_3 & \dots & x_n \\ y_1 & y_2 & y_3 & \dots & y_n \end{pmatrix}$$

Find the mean of each row. $\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$

Subtract it.

$$N = \begin{pmatrix} x_1 - \bar{x} & x_2 - \bar{x} & x_3 - \bar{x} & \dots & x_n - \bar{x} \\ y_1 - \bar{y} & y_2 - \bar{y} & y_3 - \bar{y} & \dots & y_n - \bar{y} \end{pmatrix}$$

Multiply $N * N^T$

You will get a 2x2 matrix, in which each entry is a summation over all n points. You could then divide by n

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$$c = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

Generic process

- The covariance matrix of a set of data gives the ways in which the set varies.
- The eigenvectors corresponding to the largest eigenvalues give the directions in which it varies most.

- Two applications
  - Eigenfaces
  - Time-elapsed photography

"Eigenfaces"

- Question: what are the primary ways in which faces vary?
- What happens when we apply PCA?
  - For each face, create a column vector that contains all the pixels from that face
  - This is a point in a high dimensional space (e.g., 65536 for a 256x256 pixel image)
  - Create a matrix F of all M faces in the training set.
  - Subtract off the "average face", μ, to get N
  - Compute the covariance matrix $C = N*N^T$.

$$F = \begin{bmatrix} p_{11,1} & p_{11,2} & p_{11,3} & \cdots & p_{11,M} \\ p_{12,1} & p_{12,2} & p_{12,3} & \cdots & p_{12,M} \\ p_{13,1} & p_{13,2} & p_{13,3} & \cdots & p_{13,M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{rc,1} & p_{rc,2} & p_{rc,3} & \cdots & p_{rc,M} \end{bmatrix}$$

M. Turk and A. Pentland, Eigenfaces for Recognition, *J Cog Neurosci*, 3(1)

“Eigenfaces”

- Question: what are the primary ways in which faces vary?
- What happens when we apply PCA?
  - The eigenvectors are the directions of greatest variability
  - Note that these are in 65536-D; thus form a ***face***.
  - This is an ***“eigenface”***
  - ***Here’s the first 4 from the ORL face dataset.***

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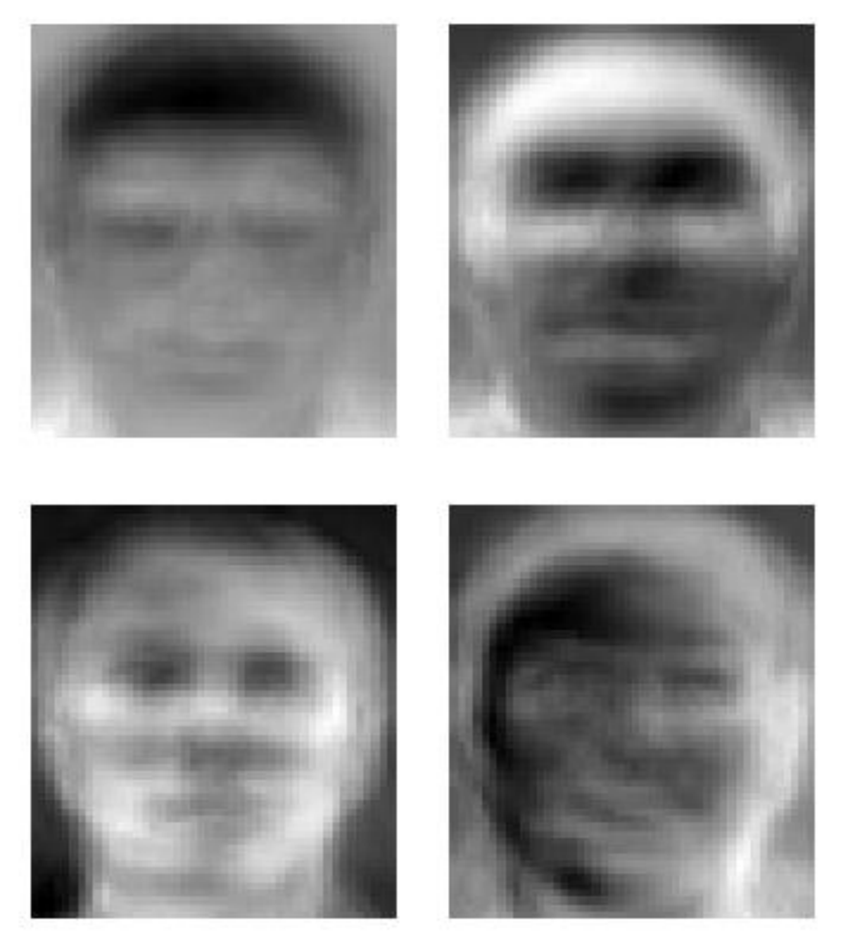
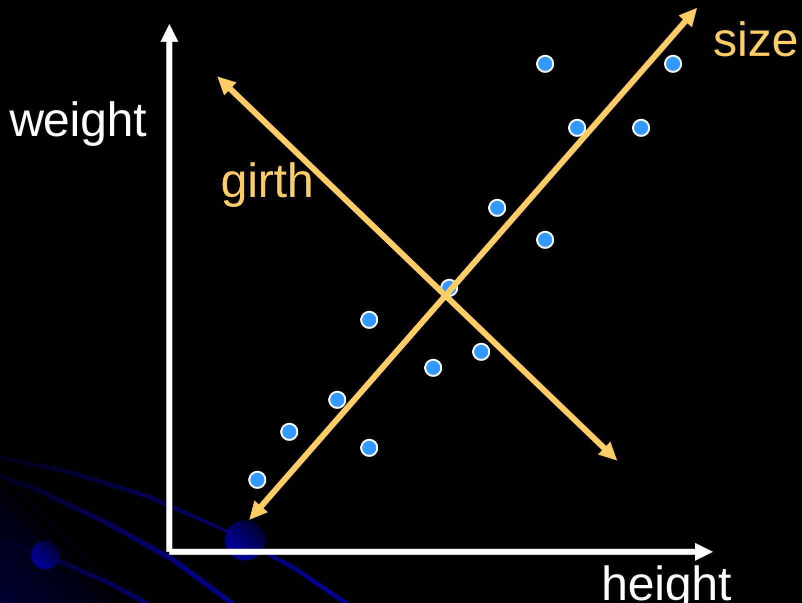


http://upload.wikimedia.org/wikipedia/commons/6/67/Eigenfaces.png; from the ORL face database, AT&T Laboratories Cambridge

Interlude: Projecting points onto lines



- We can project each point onto the principal axis. How?

Interlude: Projecting a point onto a line

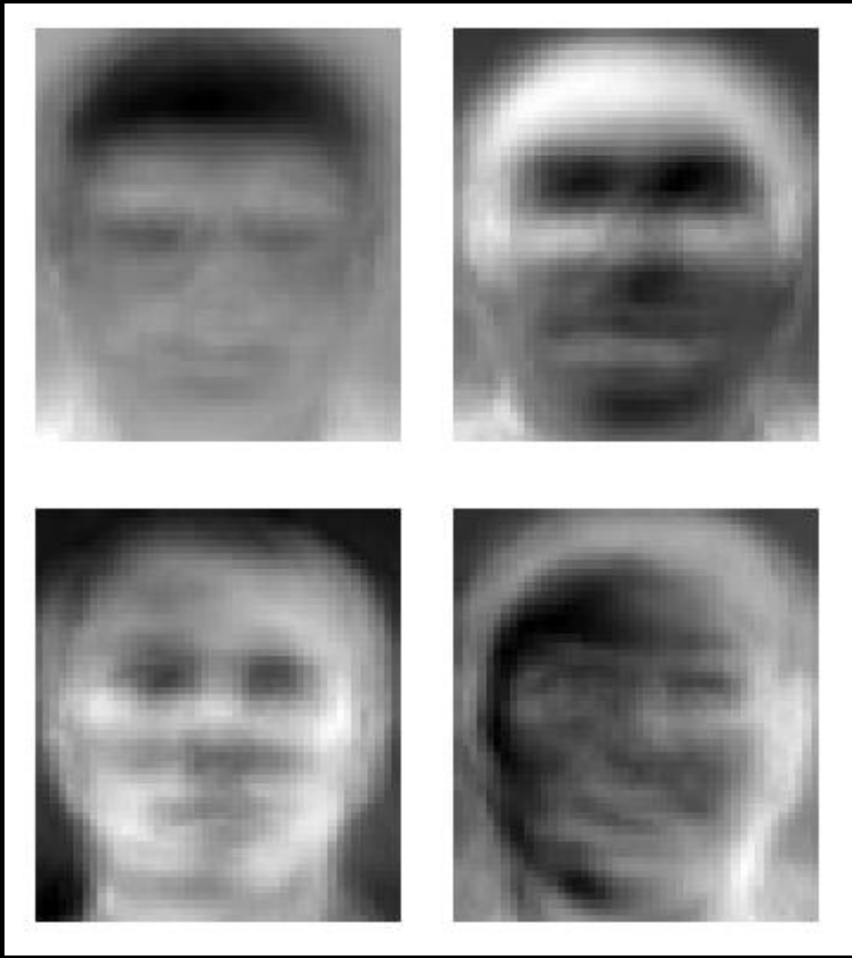
- Assuming the axis is represented by a unit vector, we can just take the dot-product of the point and the vector.
- $\mathbf{u}*\mathbf{p} = \mathbf{u}^T\mathbf{p}$ (which is 1D)
- Example: Project (5,2) onto line y=x.
- If we want to project onto two vectors, u and v simultaneously:
- Create $\mathbf{w} = [\mathbf{u}\ \mathbf{v}]$, then compute $\mathbf{w}^T\mathbf{p}$, **which is 2D.**
  - Result: p is now in terms of u and v.
  - This generalizes to arbitrary dimensions.

Application: Face detection

- If we want to project a point onto two vectors, u and v simultaneously:
- Create **w = [u v]**, then compute $\mathbf{w}^T\mathbf{p}$**, which is 2D.**
  - Result: p is now in terms of u and v.
  - This generalizes to arbitrary dimensions.
- Project a face (in 65K input space) to face space (~50 dimensions)
- A face can be approximated by a linear combination of the eigenfaces.

“Eigenfaces”

- Question: what are the primary ways in which faces vary?
- What happens when we apply PCA?
  - The top M eigenfaces for “face space”.
  - We can project any face onto these eigenvectors. Thus, any face is a linear combination of the eigenfaces.
  - Can classify faces in this lower-D space.
  - There are computational tricks to make the computation feasible



Time-elapsed photography

- Question: what are the ways that outdoor images vary over time?
- Form a matrix in which each column is an image
- Find eigs of covariance matrix

See example images on Dr. B's laptop or at the link below.

N Jacobs, N Roman, R Pless, Consistent Temporal Variations in Many Outdoor Scenes. *IEEE Computer Vision and Pattern Recognition*, Minneapolis, MN, June 2007.

Time-elapsed photography

- Question: what are the ways that outdoor images vary over time?
- The mean and top 3 eigenvectors (scaled):
- Interpretation?

N Jacobs, N Roman, R Pless, Consistent Temporal Variations in Many Outdoor Scenes. *IEEE Computer Vision and Pattern Recognition*, Minneapolis, MN, June 2007.

Time-elapsed photography

- Recall that each image in the dataset is a linear combination of the eigenimages.

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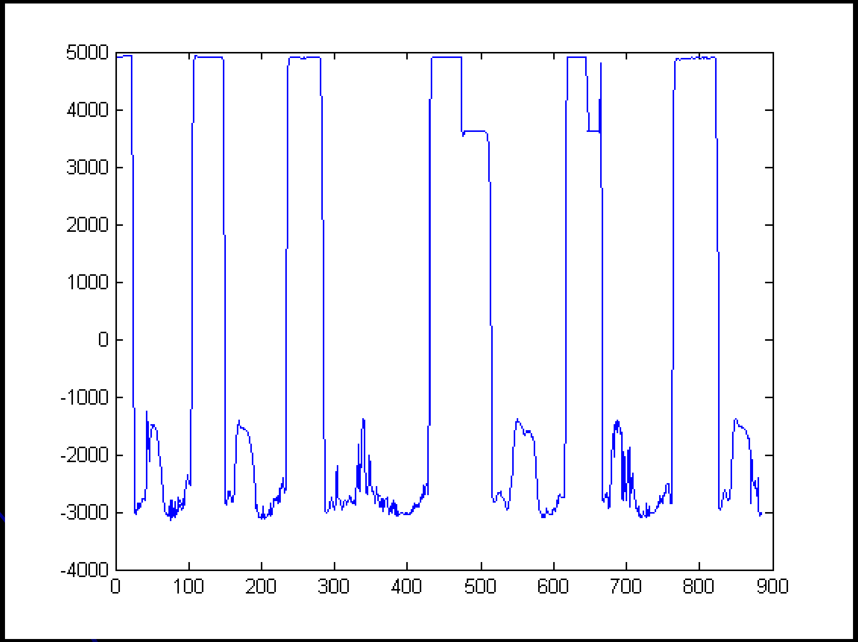
Time-elapsed photography

- Every image's projection onto the first eigenvector



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Research idea

- Done:
  - Finding the PCs
  - Using to detect latitude and longitude given images from camera
- Yet to do:
  - Classifying images based on their projection into this space, as was done for eigenfaces