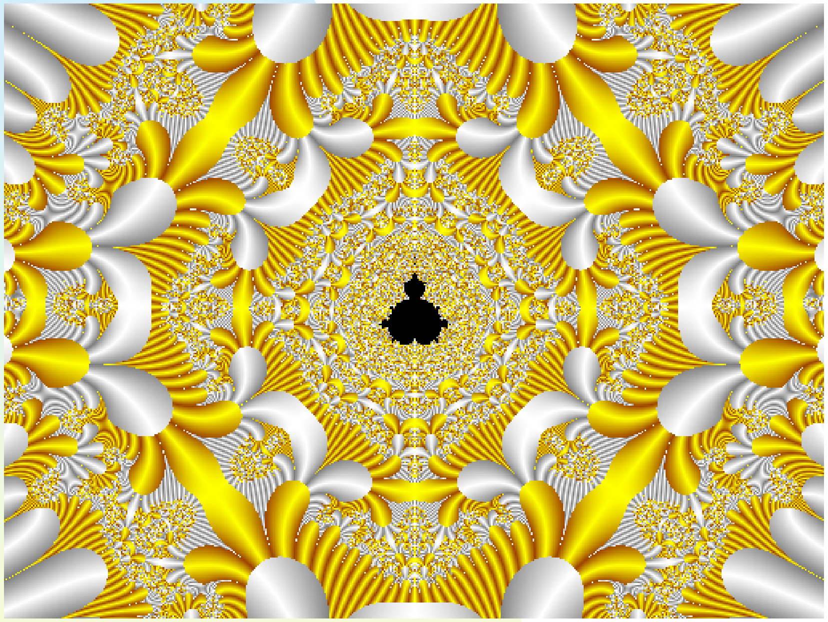
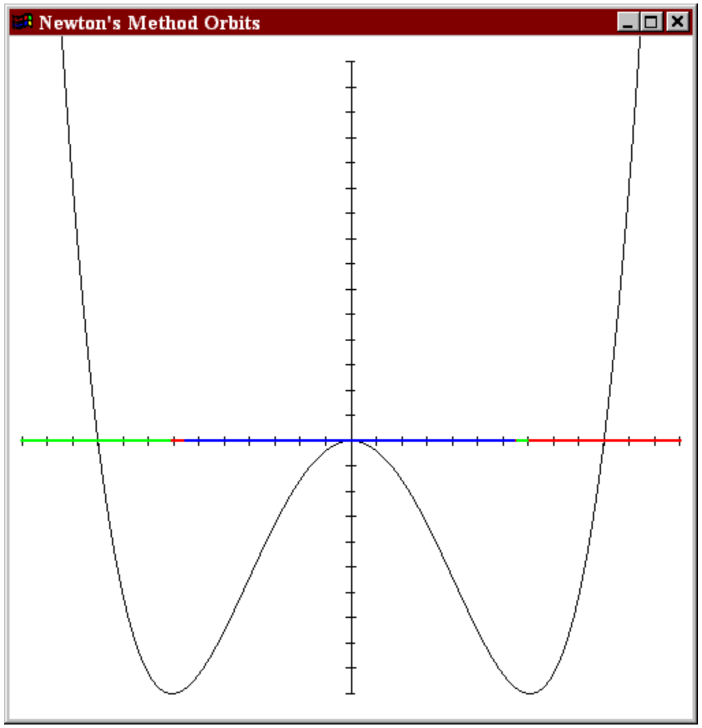
# Session overview



- Graphical analysis
- Yesterday's quiz:
  - Please look over your answer to the last question
  - Then pass it in

# Results from Newton's method study

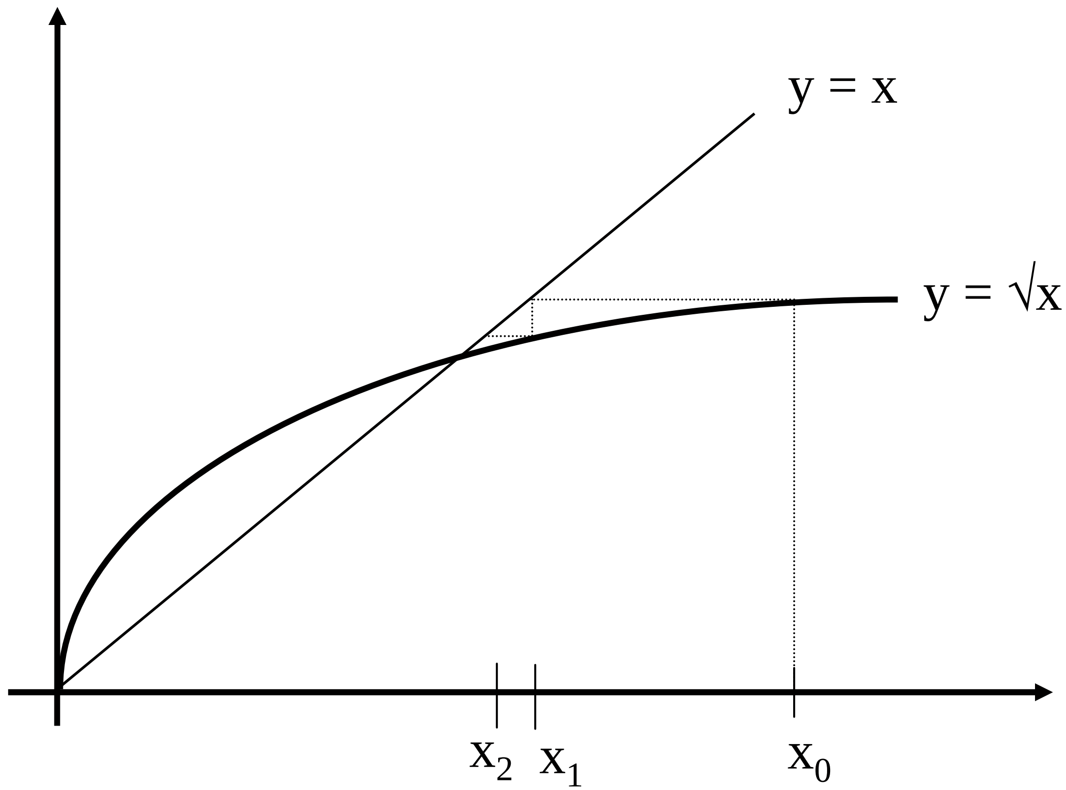
- What results did you get for the possible orbits of $4x^4-4x^2$?



# Graphical iteration

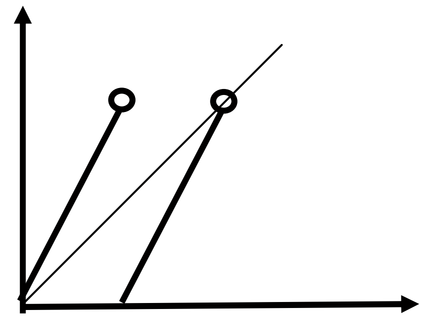
- We usually want to view iterations graphically in terms of the map itself
- In iterations, the old y value becomes the new x value
- This is accomplished graphically with the ***replacement line***, y = x
- Note that the points at which the replacement line intersects the map are the fixed points of the system

# Replacement line



# The doubling function

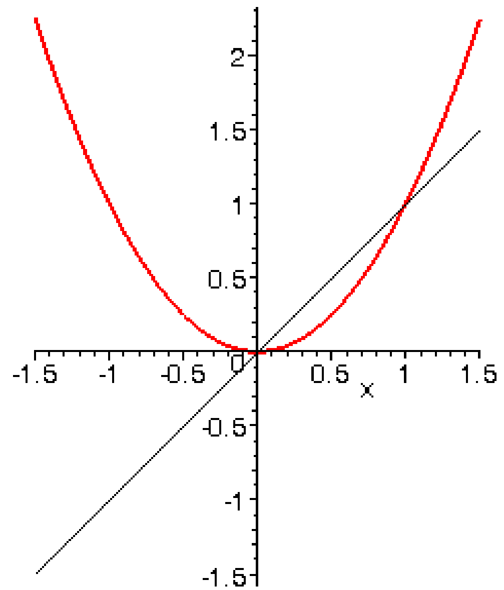
$$D(x) = \begin{cases} 2x & 0 \le x < \frac{1}{2} \\ 2x - 1 & \frac{1}{2} \le x < 1 \end{cases}$$



- D: [0, 1) $\rightarrow$ [0, 1)
- It's clear from the graph that D has one fixed point, x = 0
- Orbit of $x_0$=1/5 is { 1/5, 2/5, 4/5, 3/5, 1/5, … } so 1/5 is a period-4 point
- Graphical analysis is accomplished via these orbit diagrams

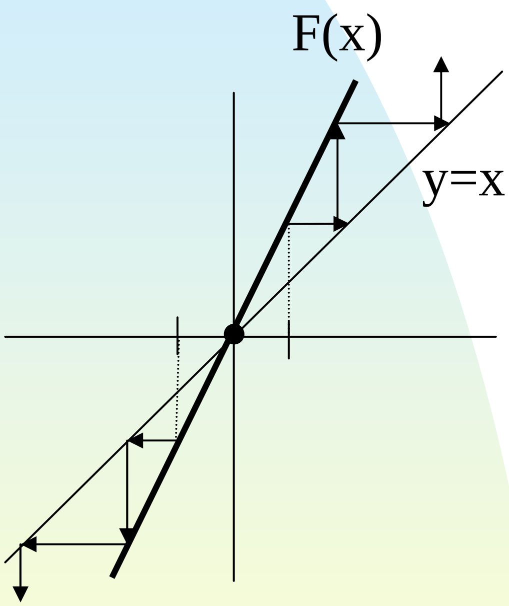
# $y = x^2$

- `plot ([x, x^2], x=-1.5..1.5, color=[black, red], thickness=[1, 2], scaling=constrained);`




- -1 is an eventually fixed point

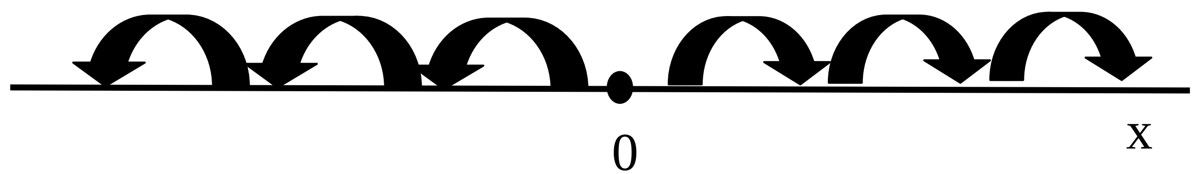
# Complete orbit analysis




- A complete description of all orbits
- Graphical analysis can be used to accomplish this
- Example: F(x) = 2x
- A complete orbit analysis:
  - x=0 is the only fixed point
  - If $x_0 < 0$, $x_n \to -\infty$ (diverges via stairstep)
  - If $x_0 > 0$, $x_n \to +\infty$ (diverges via stairstep)

# Phase portrait

- Another way to look at behavior




- Fixed points are given with a solid dot
- Arrows show the progression of a typical orbit

# Quiz

- The quiz has six function plots for which you are to do some graphical analyses
- Determine the fixed points and the behaviors of typical orbits