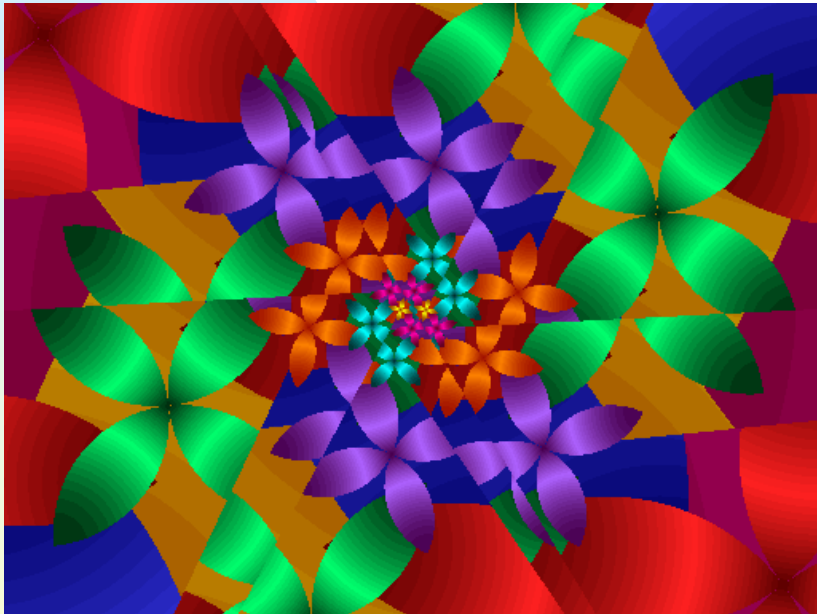


# Session overview



- Dimension
- Linear fractals program

# An approach to dimension - 1

- Consider a line segment of length 1:



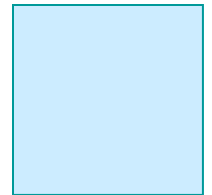
- There is one piece ( $N = 1$ ), of length 1 ( $s = 1$ ,  $1/s = 1$ )
- Now consider the same line segment divided into thirds:



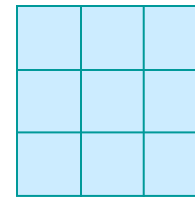
- Now there are 3 pieces ( $N = 3$ ), each of length  $1/3$  ( $s = 1/3$ ,  $1/s = 3$ )
- $3 = 3^1$

# An approach to dimension - 2

- Now consider a square of length 1:
- There is one piece ( $N = 1$ ), of length 1 ( $s = 1$ ,  $1/s = 1$ )



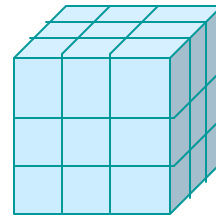
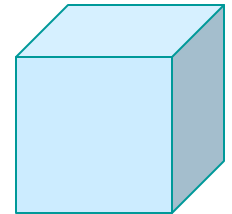
- Now consider the same square divided into thirds:



- Now there are 9 pieces ( $N = 9$ ), each of length  $1/3$  ( $s = 1/3$ ,  $1/s = 3$ )
- $9 = 3^2$

# An approach to dimension - 3

- Now consider a cube of length 1:
- There is one piece ( $N = 1$ ), of length 1 ( $s = 1$ ,  $1/s = 1$ )
- Now consider the same cube divided into thirds:



- Now there are 27 pieces ( $N = 27$ ), each of length  $1/3$  ( $s = 1/3$ ,  $1/s = 3$ )
- $27 = 3^3$

# Recap

## Dimension Relationship

$$1 \quad 3 = 3^1$$

$$2 \quad 9 = 3^2$$

$$3 \quad 27 = 3^3$$

...

N

---

In general,  $N = (1/s)^D$

# Self-similarity dimension

- From the previous slides, we observe that  $N = (1/s)^D$
- Solve for  $D$
- $\log N = \log (1/s)^D$
- $D = \log N / \log (1/s)$
- For the Koch curve,
  - ◆  $N = 4$  and  $s = 1/3$ , so  $D = 1.262$
- For the middle-thirds Cantor set,
  - ◆  $N = 2$  and  $s = 1/3$ , so  $D = 0.631$

# It's not enough

- There are many definitions of dimension
- We'll look at more later this week:
  - ◆ Box-counting
  - ◆ Hausdorff

# LinearFractals.cpp

- The `LinearFractals` program generates fractals that can be drawn with straight lines (without lifting the pen)
- Fractals are described by:
  - ◆ an initiator (level 0 of the recursion)
  - ◆ a generator to take the initiator to the first level image

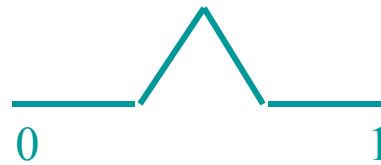


# The Koch curve

- The initiator for the Koch curve is



- The generator for the Koch curve is



- Input for `LinearFractals` for the Koch curve:

- ◆ # of pieces = 4
- ◆ contraction factor = 0.333333
- ◆ angles = 0, 60, -120, 60
- ◆ levels of recursion
- ◆ maximum time to draw

# Source code

- `LinearFractals.cpp` source code is on the ANGEL course web site and in the handout.
- Homework 0 due Thursday– make this program work on your laptop if you haven't
- Experiment with using it now
- Project 1: Modify it to draw Sierpinski's gasket.