

# FREE AND BOUND VARIABLES

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## BNF of Simple Expressions

```
<expression> ::= <identifier>  
               ::= (lambda (<identifier>) <expression>)  
               ::= (<expression> <expression>)
```

## Variable Binding

- Consider:

`(lambda (<identifier>) <expression>)`

- The occurrence of `<identifier>` is a declaration that binds all occurrences of that variable in `<expression>` unless some intervening declaration of the same variable occurs.

## Free and bound Variables

### Intuitive Definition

- A variable  $x$  occurs free in  $E$  if and only if there is some use of  $x$  in  $E$  that is not bound by any declaration of  $x$  in  $E$ .
- A variable  $x$  occurs bound in an expression  $E$  if and only if there is some use of  $x$  in  $E$  that is bound by a declaration of  $x$  in  $E$ .

## Free and bound Variables

### Example

- Example:
  - $((\mathbf{lambda} \ (\mathbf{x}) \ \mathbf{x}) \ \mathbf{y})$
- x occurs bound.
- The second occurrence of x is a reference bound by the first occurrence of x (a declaration).
- y occurs free.
- Its sole occurrence in this expression is not bound by any declaration of y.

## Occurs Free

- A variable  $x$  occurs free in a lambda calculus expression  $E$  if and only if
  1.  $E$  is a variable reference and  $E$  is the same as  $x$ ; or
  2.  $E$  is of the form  $(\mathbf{lambda} \ (\mathbf{y}) \ \mathbf{E}')$ , where  $y$  is different from  $x$  and  $x$  occurs free in  $E'$ ; or
  3.  $E$  is of the form  $(\mathbf{E1} \ \mathbf{E2})$  and  $x$  occurs free in  $E1$  or  $E2$ .

## Occurs Bound

A variable  $x$  occurs bound in a lambda calculus expression  $E$  if and only if

1.  $E$  is of the form  $(\text{lambda } (y) E')$ , where  $x$  occurs bound in  $E'$  or  $x$  and  $y$  are the same variable and  $y$  occurs free in  $E'$ ; or
2.  $E$  is of the form  $(E1 E2)$  and  $x$  occurs bound in  $E1$  or  $E2$ .

## Occurs Bound

- Examples of  $x$  Occurring Bound Based on Condition (1)
  - $(\text{lambda } (x) x)$
  - $(\text{lambda } (y) (\text{lambda } (x) x))$
  - $(\text{lambda } (x) (\text{lambda } (x) x))$
  - $(\text{lambda } (x) (\text{lambda } (y) (x y)))$