

Objectives: You should learn

- More about list processing.
- More about the use of let, letrec, named let, map, and apply.
- How to base recursive programs on recursive datatype definitions.

At the end of this document, there are several questions and answers from previous term's Piazza.

Details for these instructions are in the previous assignment, so not repeated here: Individual assignment. Comments at beginning, before each problem, and when you do anything non-obvious. Submit to server (test offline first). Your code must not mutate (unless a particular problem calls for it), read, or write anything. Assume arguments have correct form unless problem says otherwise.

Abbreviations for the textbooks: EoPL - Essentials of Programming Languages, 3rd Edition
 TSPL - The Scheme Programming Language, 4th Edition
 EoPL-1 - Essentials of Programming Languages, 1st Edition (handout)

Reading Assignment: See the schedule page. Have you been keeping up with the reading?

Problems to turn in: For many of these, you will want to write one or more helper procedures.

#1 (10 points) `vector-append-list` (`vector-append-list v ls`) returns a new vector with the elements of `ls` attached to the end of `v`. Do this without using `vector->list`, `list->vector`, or `append`.

For this problem only, you can and should use mutation: namely the `vector-set!` procedure. Note that `vector-set!` does not return a value.

Hint that I posted on Piazza for a previous term:

Try using this code, and writing the two recursive helper procedures. You are not *required* to use this code.

```
(define vector-append-list
  (lambda (vec ls)
    (let ([new-vector (make-vector (+ (vector-length vec) (length ls)))]
          (copy-from-vector new-vector vec 0)
          (copy-from-list new-vector ls (vector-length vec))
          new-vector)))
```

#2 (10 points) `group-by-two` `ls` takes a list `ls`. It returns a list of lists: the elements of `ls` in groups of two. If `ls` has an odd number of elements, the last sublist of the return value will have one element.

group-by-two: List → ListOf(List)

```
> (group-by-two '())
()
> (group-by-two '(a))
((a))
> (group-by-two '(a b))
((a b))
> (group-by-two '(a b c))
((a b) (c))
> (group-by-two '(a b c d e f g))
((a b) (c d) (e f) (g))
> (group-by-two '(a b c d e f g h))
((a b) (c d) (e f) (g h))
```

#3 (20 points) `group-by-n` `ls` `n` takes a list `ls` and an integer `n` (you may assume that $n \geq 2$). Returns a list of lists: the elements of `ls` in groups of `n`. If `ls` has a number of elements that is not a multiple of `n`, the length of the last sublist of the return value will be less than `n`. **For full credit, your code must run in time $O(\text{length}(ls))$. In particular, this means that no recursive procedure in your code can call `(length ls)`.**

group-by-n: List → ListOf(List)

```

> (group-by-n '() 3)
()
> (group-by-n '(a b c d e f g) 3)
((a b c) (d e f) (g))
> (group-by-n '(a b c d e f g) 4)
((a b c d) (e f g))
> (group-by-n '(a b c d e f g h) 4)
((a b c d) (e f g h))
> (group-by-n '(a b c d e f g h i j k l m n o) 7)
((a b c d e f g) (h i j k l m n) (o))
> (group-by-n '(a b c d e f g h) 17)
((a b c d e f g h))
> (group-by-n '(a b c d e f g h i j k l m n o p q r s t) 17)
((a b c d e f g h i j k l m n o p q) (r s t))

```

#4 (57 points) Consider the following syntax definition from page 9 of EoPL:

```
<bintree> ::= <integer> | ( <symbol> <bintree> <bintree> )
```

Note that this representation is quite different than the BST representation in Assignment 6.

Write the following procedures:

- (bt-leaf-sum T) finds the sum of all of the numbers in the leaves of the bintree T.
- (bt-inorder-list T) creates a list of the symbols from the *interior* nodes of T, in the order that they would be visited by an inorder traversal of the binary tree.
- (bt-max T) returns the largest integer in the tree.
- (bt-max-interior T) takes a binary tree with at least one interior node, and returns (in O(N) time, where N is the number of nodes) the symbol associated with an interior node whose subtree has a maximal leaf sum (at least as large as the sum from any other interior node in the tree). If multiple nodes in the tree have the same maximal leaf-sum, return the symbol associated with the leftmost (as it appears in the tree's printed representation) maximal node.

This bt-max-interior procedure is trickier than it looks at first!

- You may not use mutation.
- You may not traverse any subtree twice (such as by calling bt-leaf-sum on every interior node).
- You may not create any additional size- $\Omega(N)$ data structures that you then traverse to get the answer.
- Think about how to return enough info from each recursive call to solve this without doing another traversal.

Note: We will revisit this linear-time bt-max-interior problem several times during this course. If you do not get this version, the later versions will be harder for you, so you should do what it takes to get this one.

#5 (50 points)

These s-list procedures have a lot in common with the s-list [procedures](#) that we wrote during our Session 8 class. Recall the extended BNF grammar for s-lists:

```

<s-list>          ::= ( {<s-expression>}* )
<s-expression> ::= <symbol> | <s-list>

```

FOLLOW THE GRAMMAR!

(a) (slist-map proc slist) applies proc to each element of slist.

slist-map: *procedure* \times *Slist* \rightarrow *NestedListOfThingsThatAreInTheRangeOfProcedure*

```

(slist-map symbol? '((a (()) b) c () (d e)))       $\rightarrow$  ((#t (()) #t) #t () (#t #t))

(slist-map (lambda (x)
  (let ([s (symbol→string x)])
    (string→symbol (string-append s s))))
  '((b (c) d) e ((a)) () e))       $\rightarrow$  ((bb (cc) dd) ee ((aa)) () ee)

```

(b) (slist-reverse slist) reverses slist and all of its sublists.

slist-reverse: *Slist* \rightarrow *Slist*

```
(slist-reverse '(a (b c) ( ) (d (e f)))) → (((f e) d) ( ) (c b) a)
```

- (c) (`slist-paren-count slist`) counts the number of parentheses required to produce the printed representation of `slist`. You must do this by traversing the structure, not by having Scheme give you a string representation of the list and counting parenthesis characters. You can get this count by looking at `cars` and `cdrs` of `slist`).

slist-paren-count: *Slist* → *Integer*

```
(slist-paren-count '()) → 2
(slist-paren-count '(a (b c) d)) → 4
(slist-paren-count '(a (b) (c ( ) ((d)))))) → 12
```

Note: s-lists are always *proper* lists.

- (d) (`slist-depth slist`) finds the maximum nesting-level of parentheses in the printed representation of `slist`. You must do this by traversing the structure, and *not* by having Scheme give you a string representation of the list and counting the maximum nesting of parenthesis characters.

slist-depth: *Slist* → *Integer*

```
(slist-depth '()) → 1
(slist-depth '(a b c)) → 1
(slist-depth '(a (b c) d)) → 2
(slist-depth '(a (b (c)) (a b))) → 3
(slist-depth '(((a) ( ) b) (c d) e)) → 4
```

- (e) (`slist-symbols-at-depth slist d`) returns a list of the symbols from `slist` whose depth is the positive integer `d`. They should appear in the same order in the return list as in the original s-list. This one has the basic pattern of the other s-list procedures, but when writing the solution, I found it easier to use a slight variation on that pattern.

slist-symbols-at-depth: *Slist* × *PositiveInteger* → *ListOf(Symbol)*

```
(slist-symbols-at-depth '(a (b c) d) 2) → (b c)
(slist-symbols-at-depth '(a (b c) d) 1) → (a d)
(slist-symbols-at-depth '(a (b c) d) 3) → ()
```

#6 (10 points) (`path-to slist sym`) produces a list of `cars` and `cdrs` that (when read left-to-right) take us to the position of the leftmost occurrence of `sym` in the s-list `slist`. Notice that the returned list contains the symbols `'car` and `'cdr`, not the `car` and `cdr` procedures. Return `#f` if `sym` is not in `slist`. Only traverse as much of `slist` as is necessary to find `sym` if it is there.

```
> (path-to '(a b) 'a)
(car)
> (path-to '(c a b) 'a)
(cdr car)
> (path-to '(c ( ) ((a b))) 'a)
(cdr cdr car car car)
> (path-to '(((d (f ((b a)) g))) 'a)
(car cdr car cdr car car cdr car)
> (path-to '(((d (f ((b a)) g))) 'c)
#f
```

#7 (25 points) Predefined Scheme procedures like `cadr` and `cdadr` are compositions of up to four `cars` and `cdrs`. You are to write a generalization called `make-c...r`, which does the composition of any number of `cars` and `cdrs`. It takes one argument, a string of a's and d's, which are used like the a's and d's in the names of the pre-defined `c...r` functions. For example, `(make-c...r "addddd")` is equivalent to `(compose car cdr cdr cdr cdr)`.

```
> (define caddddr (make-c...r "addddd"))
> (caddddr '(a (b) (c) (d) (e) (f)))
(e)
> ((make-c...r "") '(a b c))
(a b c)
> ((make-c...r "a") '(a b c))
a
> ((make-c...r "ddaddd") '(a b c ((d e f g) h i j)))
(i j)
> ((make-c...r "addddddddddd") '(a b c d e f g h i j k l m))
l
```

I have provided the code for `compose`. We will discuss this solution in class soon. For now you can just use it here. For full credit, you should write `make-c...r` in a functional style that only applies (calls)

- built-in procedures (including, `car`, `cdr`, `map`, and `apply`, of course),
- anonymous procedures, and
- `compose` (which I am giving you, so it does not count as a self-written recursive procedure)

in your definition of `make-c...r`. More precisely, you may not write and call your own recursive procedures

```
(define compose
  (case-lambda
    [() (lambda (x) x)]
    [(first . rest)
     (let ([composed-rest (apply compose rest)])
       (lambda (x) (first (composed-rest x))))]))
```

Hint: My solution uses the built-in procedures `map`, `apply`, `string->list`, `list->string`, `string->symbol`, and `eval`; also the character constants `#\c` and `#\r`. You are not required to use these, but you may find them helpful. I do not assume that you are already familiar with all of them; *The Scheme Programming Language* contains info on them; my intention is that you demonstrate an ability to look up and use new procedures. My solution calls `map` multiple times.

Piazza posts related to this assignment from previous terms:

A7 #1 vector-append-list

Here is an example that has some things in common with `vector-append-list`. It makes a new vector that is the reverse of it's argument. This procedure is not directly useful for problem #1, but it may help you to better understand the use if vectors.

```
(define vector-reverse ; return a vector that is the reverse of v
  (lambda (v)
    (let* ([v-len (vector-length v)]
           [result (make-vector v-len #f)])
      (let loop ([i 0])
        (if (< i v-len) ; one-armed if, has no "else" part.
            (begin (vector-set! result (- v-len i) 1)
                    (vector-ref v i)
                    (loop (+ i 1))))
            result))))

(vector-reverse '#(2 3 4 5))
```

Are we required to use vector-set! for our solution? As it's not too difficult to have a solution making use of the vector procedure instead.



Claude Anderson 5 months ago

No, you are not required to use vector-set!

Most Scheme mutation procedures (such as vector-set!) do not return a value.

The subject line says it all.

A7 Bintree internal sum

Is it allowed if we return the sum for each internal node and just find the max of those sums? It only adds another $O(n)$ to find the max.

the instructors' answer,

where instructors collectively construct a single answer

But it bypasses what I wanted you to figure out how to do, so no.

group-by-two and group-by-n: Don't be afraid to apply reverse in your code

I talked with a student today who was reluctant to use reverse, thinking that I have discouraged its use. I think it may be easiest and most efficient to write the group-by procedures using cons, and then reversing the list once you have all of it. Calling reverse once is $O(N)$. Appending each element to the end of a list (one at a time) is $O(N^2)$.

Seeing the procedure that compose makes

I'm writing make-c...r, and am having a hard time debugging since tracing the function just gives #<procedure>, so I can't really see what's going wrong with the composing.

Is there some way to see the function that compose makes or if not other ways to debug it?

Edit: right now I'm at the point where I'm composing a list of cars and cdrs (ex: '(car cdr cdr)) , and would be helped if I could see what the result of composing them looked like

the students' answer,

where students collectively construct a single answer

You may be able to trace compose itself.

the instructors' answer,

where instructors collectively construct a single answer

Actions

trace-lambda may be what you need

(trace-lambda trace-name (args) bodies), where trace-name is a name that you make up so that the tracing can identify which procedure is being traced.

There are also trace-let, trace-letrec, and trace-define.