Grammar for lambda-calculus expresions:

<LcExpr> ::=	<identifier> |	**variable use**
	(lambda (<identifier>) <LcExpr>) |	**abstraction**
	(<LcExpr> <LcExpr>)	**application**

CSSE 304 Day 12

1. **Free and bound Examples:** In each of the following expressions, does *x* occur free and/or occur bound?

   a) x

   b) t

   c) (x t)

   d) (lambda (x) (x t))

   e) ((lambda (x) x) x)

   f) (lambda (x) (lambda (t) (t x)))

Variable *x* **occurs free** in the LcExp *e* iff one of the following is true:

- **F1.** *e* is a variable, and *e* is the same as *x*.
- **F2.** *e* is an abstraction (λ (y) e'), where *y* is different from *x* and *x* occurs free in *e*'.
- **F3.** *e* is an application (e_1 e_2), where x occurs free in e_1 or in e_2.

Variable *x* **occurs bound** in the LcExp *e* iff one of the following is true:

- **B1.** *e* is an abstraction (λ (y) e'), where *x* occurs bound in *e*', **or** *x* and *y* are the same variable and *x* occurs free in *e*'.
- **B2.** *e* is an application (e_1 e_2) where x occurs bound in e_1 or in e_2.

```
(define occurs-bound?
    (lambda (sym exp)
       (cond
```

2. The **lexical depth** of a bound occurrence of a variable is the number of levels of nested `lambda`s and `let`s between this occurrence and the variable's definition. In

```
(lambda (z)
  (lambda (x)
    (lambda (y) (x y)))),
```

the occurrence of *y* has depth 0 and the occurrence of *x* has depth 1. There is no occurrence of z.

3. The **lexical address** of a bound occurrence of a variable is a pair `(d p)`, where d is that occurrence's lexical depth, and p is the variable's position within its "declaration list". The lexical address of a free variable includes the variable's name and an indication that it is free.

4. Example:

   In **(lambda (x z)**
   (lambda (y)
   ((x y) z)))

The occurrence of x has depth 1 and position 0.
The occurrence of y has depth 0 and position 0.
The occurrence of z has depth 1 and position 1.

5. Example of output from the lexical-address procedure that you will write:

```
(lexical-address '(lambda (a b c)
                    (if (eq? b c)
                        ((lambda (c)
                           (cons a c))
                         a)
                        b)))
```

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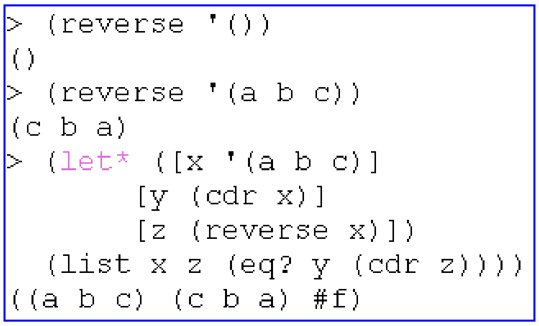
```
(lambda (a b c)
  (if ((: free eq?) (: 0 1) (: 0 2))
      ((lambda (c)
        ((: free cons)  (: 1 0) (: 0 0)))
       (: 0 0))
      (: 0 1)))
```

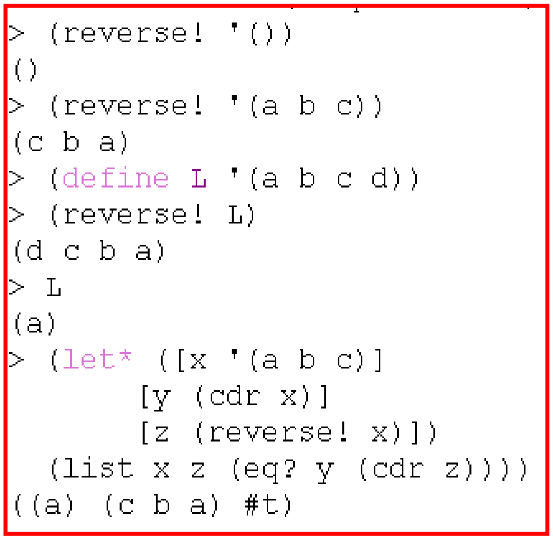
6. Lexical address exercises (also see example that comes before these in the slides).

```
(lexical-address
 '((lambda (x y)
  (((lambda (z)
      (lambda (w y)
        (+ x z w y)))
    (list w x y z))
   (+ x y z)))
(y z)))
```

```
(lexical-address
 '(let ([a 3] [b 4])
    (let ([a (+ b 2)] [c a])
               (+ a b c))))
```



```
> (reverse '())
()
> (reverse '(a b c))
(c b a)
> (let* ([x '(a b c)]
         [y (cdr x)]
         [z (reverse x)])
    (list x z (eq? y (cdr z))))
((a b c) (c b a) #f)
```



```
> (reverse! '())
()
> (reverse! '(a b c))
(c b a)
> (define L '(a b c d))
> (reverse! L)
(d c b a)
> L
(a)
> (let* ([x '(a b c)]
         [y (cdr x)]
         [z (reverse! x)])
    (list x z (eq? y (cdr z))))
((a) (c b a) #t)
```