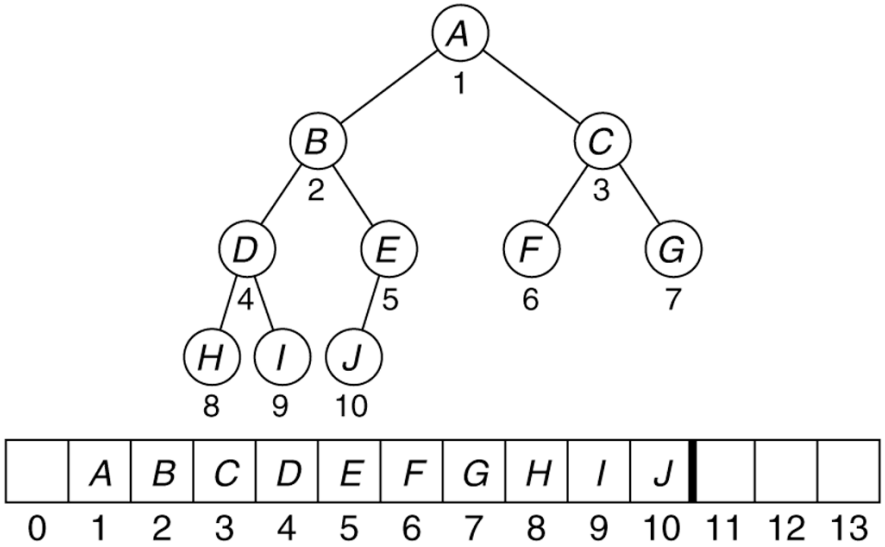


CSSE 230 Day 20

Priority Queues
Heaps

After this lesson, you should be able to …
… apply the binary heap insertion and deletion algorithms by hand
… implement the binary heap insertion and deletion algorithms

Exam 3

- Format same as Exam 1 (written and programming)
  - One 8.5x11 sheet of paper (one side) for written part
  - Same resources as before for programming part
- Topics: weeks 1-7
  - Through day 21, HW7, and EditorTrees milestone 3
  - Especially Binary trees, including BST, AVL, indexed (EditorTrees), red-black*
  - Traversals and iterators, size vs. height, rank
  - Recursive methods, including ones that should only touch each node once for efficiency (like sum of heights and isHeightBalanced from HW5)
  - Hash tables
  - Heaps - basic concepts (we won't ask you to write code yet)
- Practice exam posted in Moodle and code in repos
- * Red-black tree coverage depends on instructor

Priority Queue ADT

Basic operations

Implementation options

Priority Queue operations

- Each element in the PQ has an associated **priority**
  - Could be specified (extra parameter) when an item is inserted into the PQ: insert(item, priority)
  - More commonly, priority is inferred from the comparable type (in our examples, an integer).
- Operations:
  - insert(item) (also called add/offer)
  - findMin()
  - deleteMin( ) (also called remove/poll)
  - isEmpty() …

Priority queue implementation

- Can we reasonably implement PQ using data structures that we already know about?
  - Array?
  - Sorted array?
  - AVL?
- One efficient approach uses a binary heap
  - A complete binary tree that is ordered in a special way
- Questions we'll ask:
  - How can we efficiently represent a complete binary tree?
  - Can we add and remove items efficiently without destroying the "heapness" of the structure?

Binary Heap

An efficient implementation of the PriorityQueue ADT

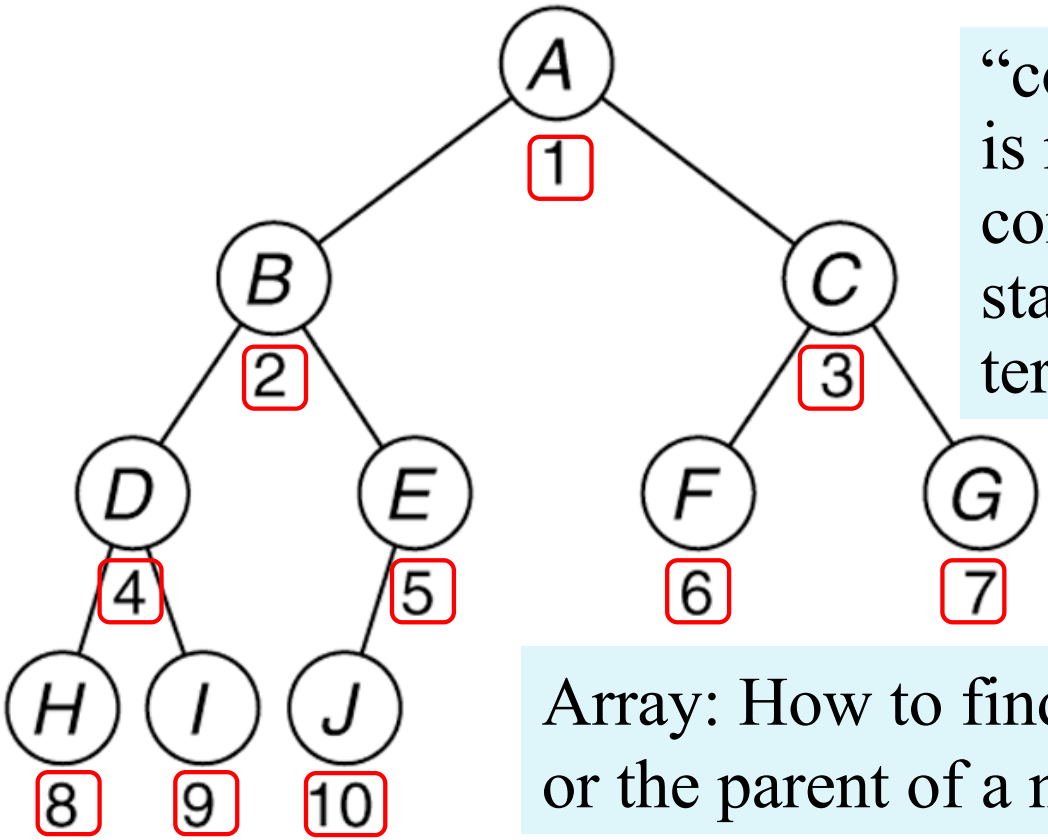
Storage (an array)

Algorithms for insertion and deleteMin

Figure 21.1

A *complete binary tree* and its array representation

Notice the lack of explicit pointers in the array



"complete" is not a completely standard term

Array: How to find the children or the parent of a node?

One "wasted" array position (0)

The (min) heap-order property: every node's value is ≤ its childrens' values

$P \leq X$

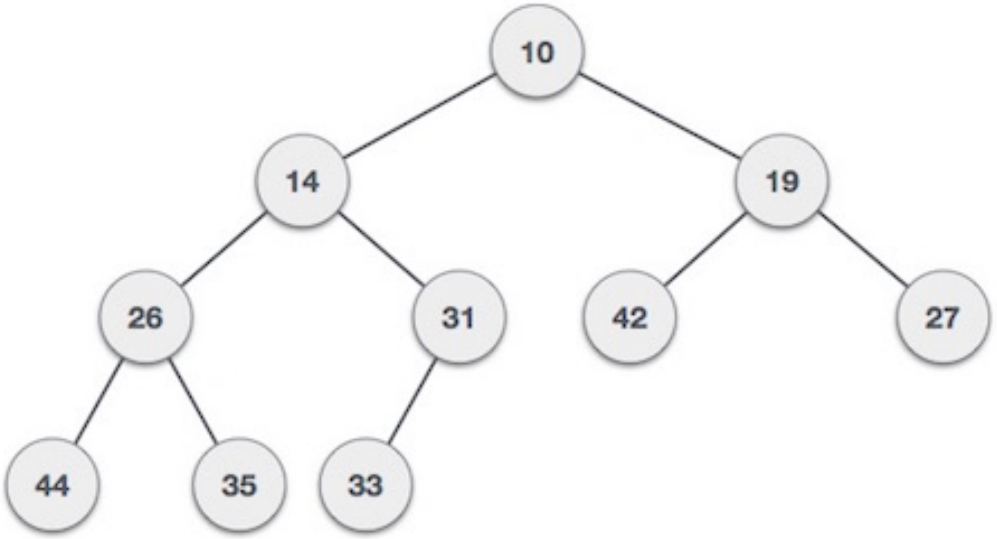
A **Binary** (min) **Heap** is a complete Binary Tree (using the array implementation, as on the previous slide) that has the heap-order property everywhere.

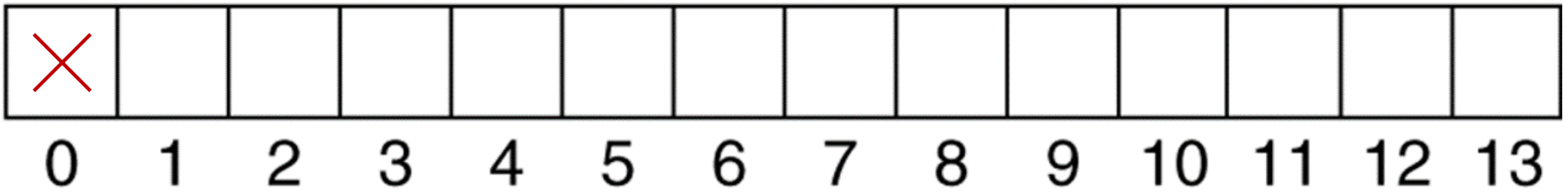
Q: In a binary heap, where do we find
- The smallest element?
- 2nd smallest?
- 3rd smallest?

Correspondence – Abstract Heap to Array Rep

Fill in the array with values from the min-heap

- Heap size = # items in the heap
- Array capacity = size of the array





Insert and DeleteMin

Idea of each:

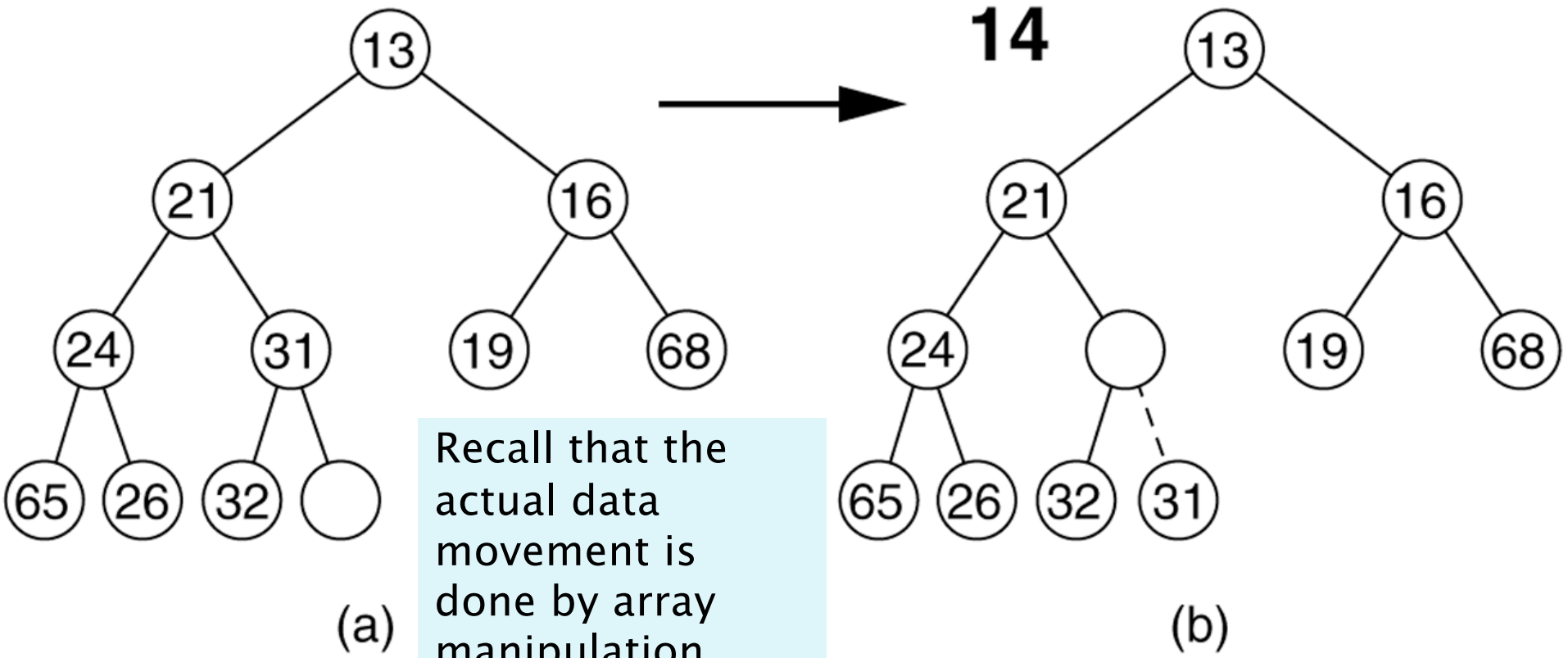
1. Get the **structure** right first
   - Insert at end (bottom of tree)
   - Move the last element to the root after deleting the root
2. Restore the heap-order property by percolating (swapping an element/child pair)
   - Insert by percolating *up*: swap with parent
   - DeleteMin by percolating *down*: swap with child with min value

Nice demo:
http://www.cs.usfca.edu/~galles/visualization/Heap.html

Figure 21.7

Attempt to insert 14, creating the hole and bubbling the hole up

Insertion algorithm



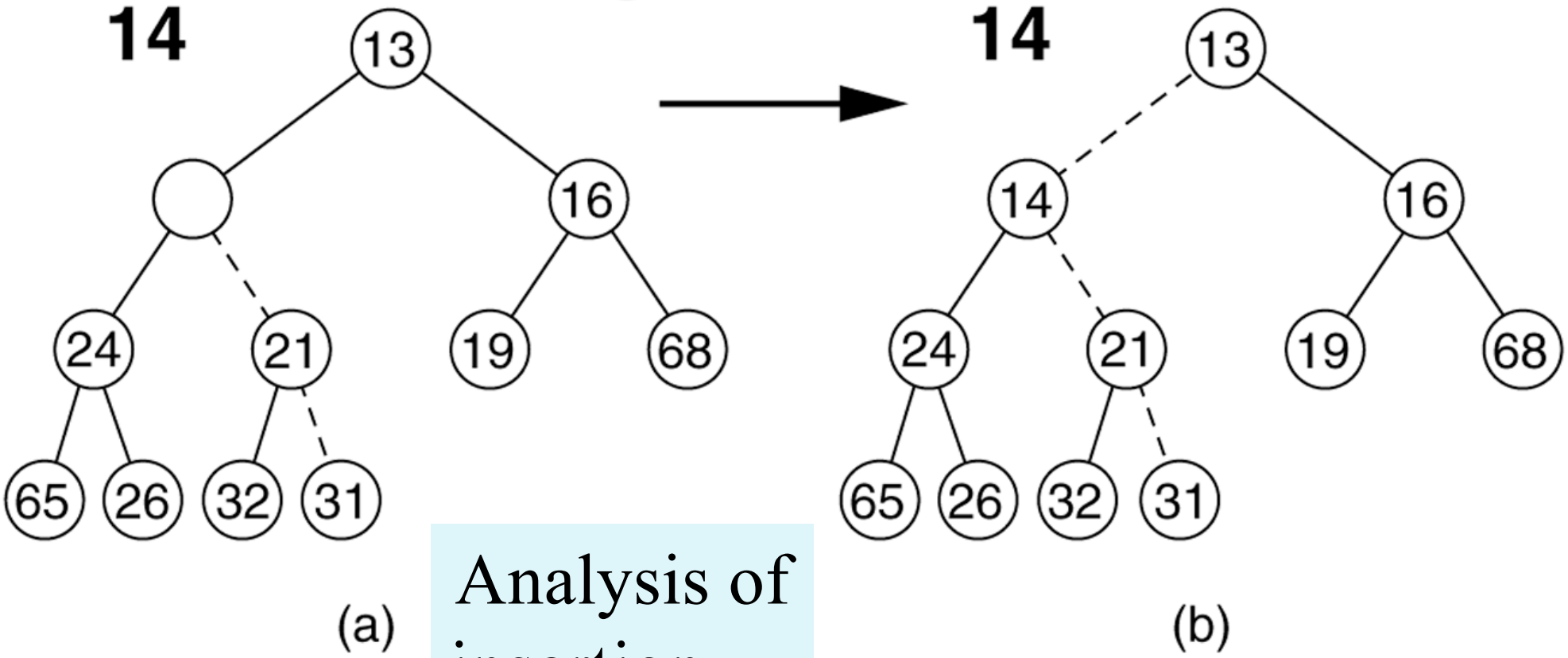
Recall that the actual data movement is done by array manipulation

Create a "hole" where 14 can be inserted.
Percolate up!

Figure 21.8

The remaining two steps required to insert 14 in the original heap shown in Figure 21.7

Insertion Algorithm continued



Analysis of insertion …

Code for Insertion

```
1       /**
2        * Adds an item to this PriorityQueue.
3        * @param x any object.
4        * @return true.
5        */
6       public boolean add( AnyType x )
7       {
8           if( currentSize + 1 == array.length )
9               doubleArray( );
10
11              // Percolate up
12          int hole = ++currentSize;
13          array[ 0 ] = x;
14
15          for( ; compare( x, array[ hole / 2 ] ) < 0; hole /= 2 )
16              array[ hole ] = array[ hole / 2 ];
17          array[ hole ] = x;
18
19          return true;
20      }
```

figure 21.9

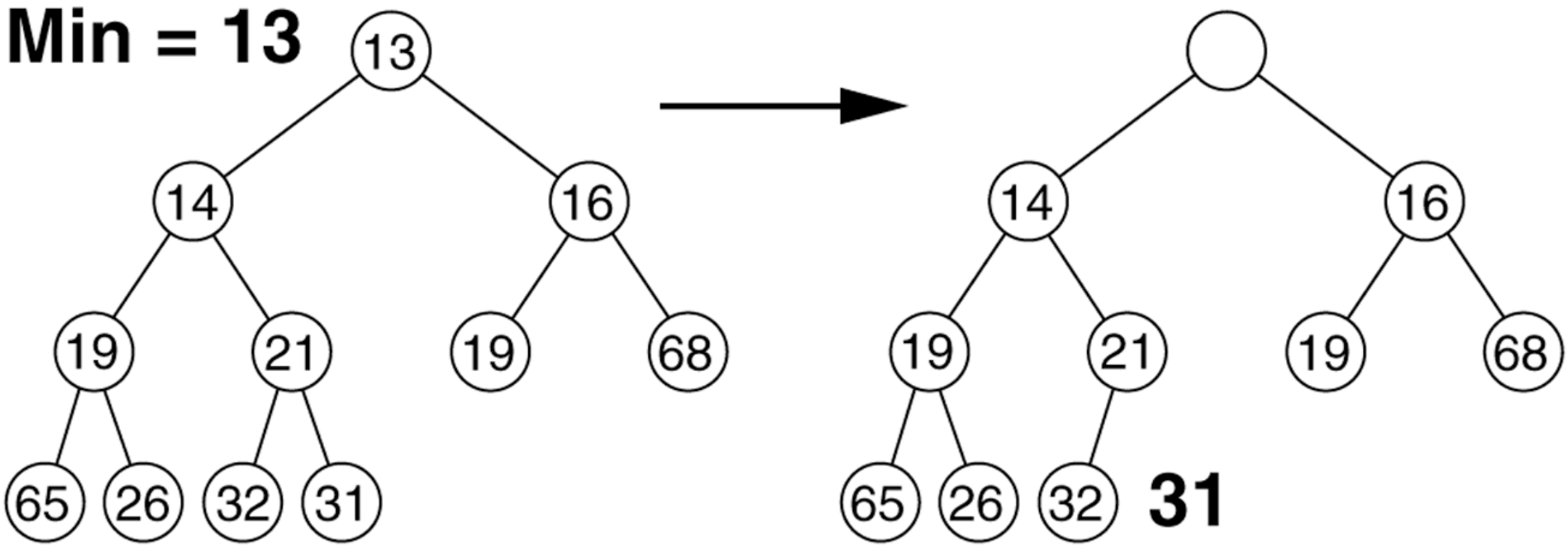
The add method

Your turn:

1. Draw an empty array representation
2. Insert into an initially empty heap: 6 4 8 1 5 3 2 7

DeleteMin algorithm

The *min* is at the root. Delete it, then use the **percolateDown** algorithm to find the correct place for its replacement.



We must decide which child to promote, to make room for 31.

Figure 21.10 Creation of the hole at the root

Figure 21.11
The next two steps in the deleteMin operation

DeleteMin Slide 2

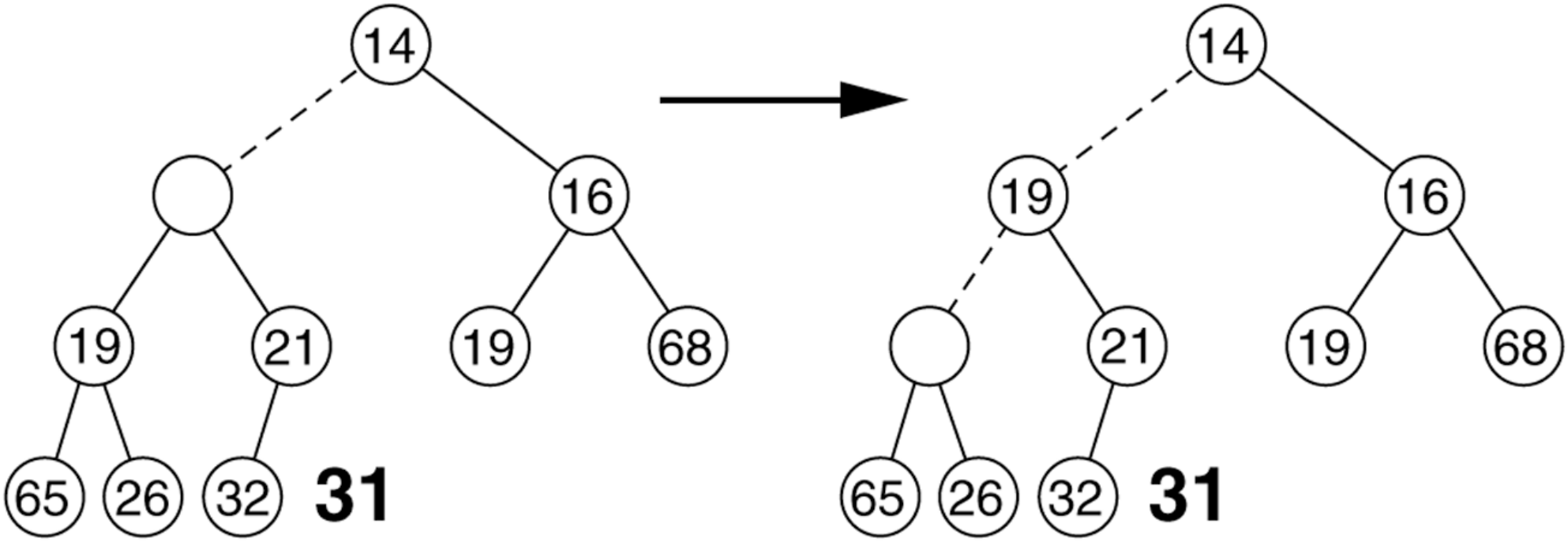
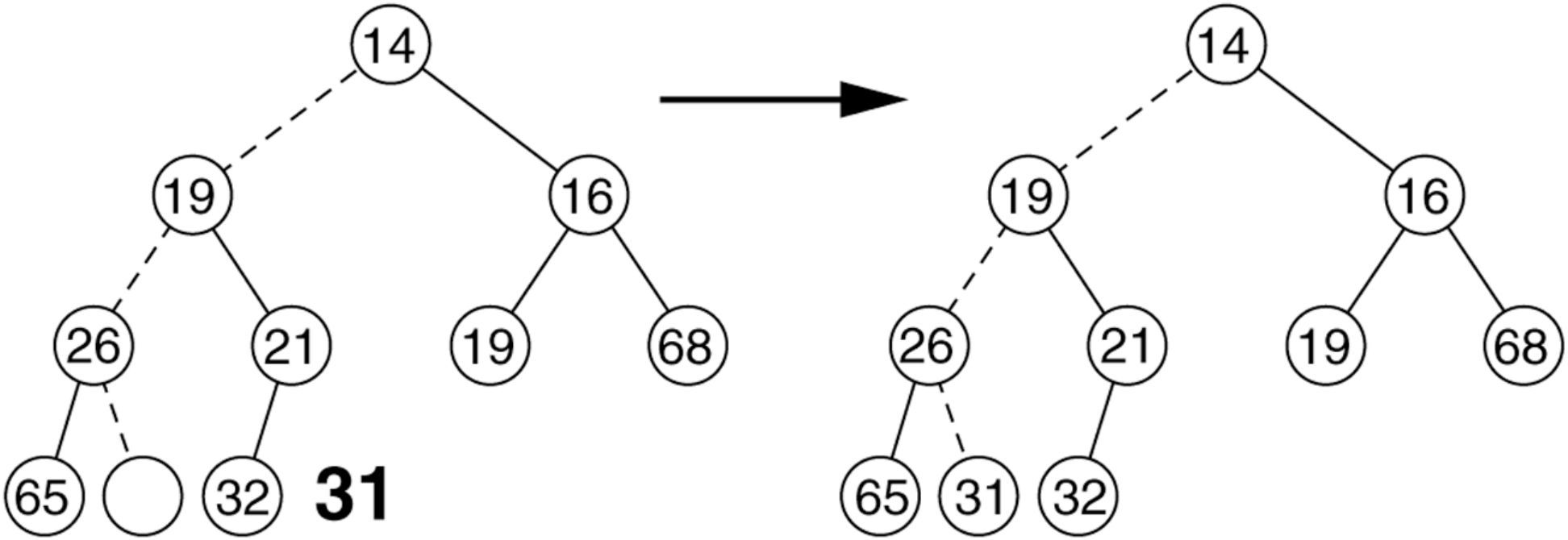


Figure 21.12
The last two steps in the deleteMin operation

DeleteMin Slide 3




```
public Comparable deleteMin( )
{
    Comparable minItem = findMin( );
    array[ 1 ] = array[ currentSize-- ];
    percolateDown( 1 );

    return minItem;
}
```

```
private void percolateDown( int hole )
{
    int child;
    Comparable tmp = array[ hole ];

    for( ; hole * 2 <= currentSize; hole = child )
    {
        child = hole * 2;
        if( child != currentSize &&
                array[ child + 1 ].compareTo( array[ child ] ) < 0 )
            child++;
        if( array[ child ].compareTo( tmp ) < 0 )
            array[ hole ] = array[ child ];
        else
            break;
    }
    array[ hole ] = tmp;
}
```

Compare node to its children, moving root down and promoting the smaller child until proper place is found.

We'll re-use percolateDown in HeapSort

Insert and DeleteMin commonalities

Idea of each:

1. Get the **structure** right first
   - Insert at end (bottom of tree)
   - Move the last element to the root after deleting the root
2. Restore the heap-order property by percolating (swapping an element/child pair)
   - Insert by percolating *up*: swap with parent
   - Delete by percolating *down*: swap with child with min value

Summary: Implementing a Priority Queue as a binary heap

- Worst case times:
  - findMin: O(1)
  - insert: amortized O(log n), worst O(n)
  - deleteMin O(log n)

- Big-O (amortized) times for insert/delete are the same as for balanced BSTs, but ..
  - Heap operations are *much* simpler to write.
  - A heap doesn't require additional space for pointers, balance codes or R-B colors, etc.
  - BinaryHeap findMin is faster.