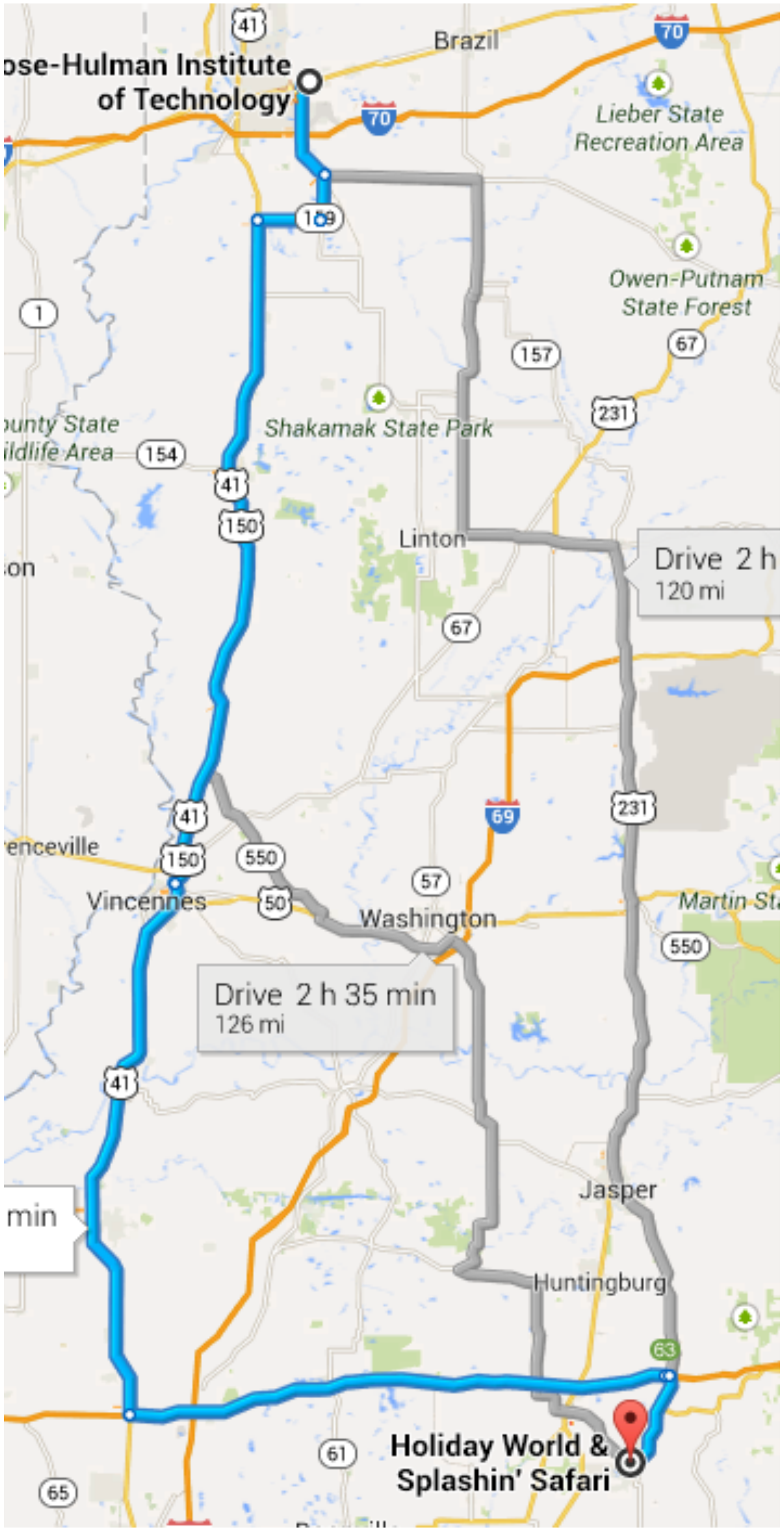
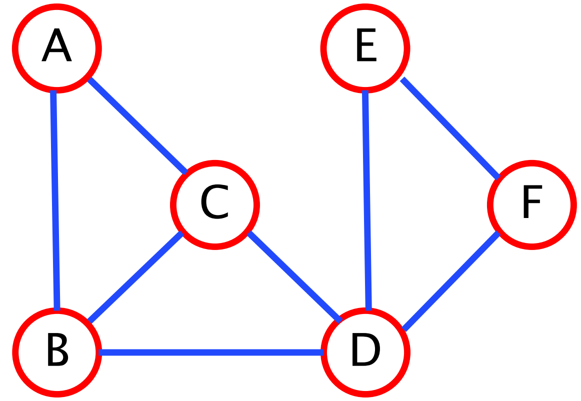


Review



CSSE 230 Day 22

What do graphs have that trees don't?

After this lesson, you should be able to …
… define the major terminology relating to graphs
… implement a graph in code, using various conventions

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Graphs

Terminology
Representations
Algorithms

A former student talks about how useful graphs are on the job!

I received this unsolicited email from former student, Sang Choi, in June 2020:

"It's literally one day into my summer vacation and **I'm already using 230 content in my work.** My research internship started yesterday, and my research project heavily involves implementing formal methods for the verification of programs. I've been assigned to read a lot of papers on this stuff, and I've noticed that **formal methods are graph-based**. There's states and transitions, which are vertices and edges in graphs. The paper I'm currently reading also goes into depth about how an increase of states in the model of a device might create a greater time complexity of O(N^2) but the author found out it was an O(N) increase after some testing .... When he runs these tests, he also talks about how many vertices he's visited, and how many minutes it took him to run the test. This just shows **how important data structures are to computer science**."

Graph Definitions

A graph G = (V,E) is composed of:

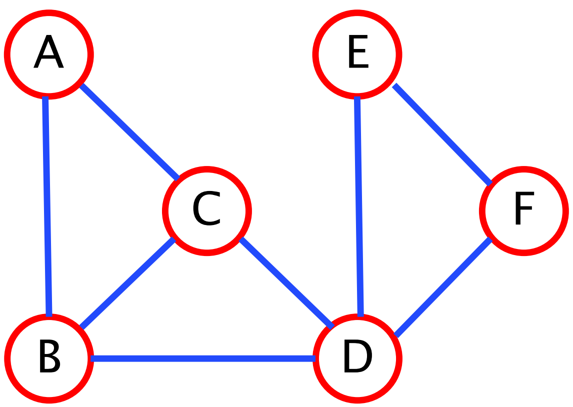
V: set of *vertices* (singular: vertex)

E: set of *edges*

An edge is a pair of vertices. Can be

unordered: $e = \{u,v\}$ (*undirected* graph)

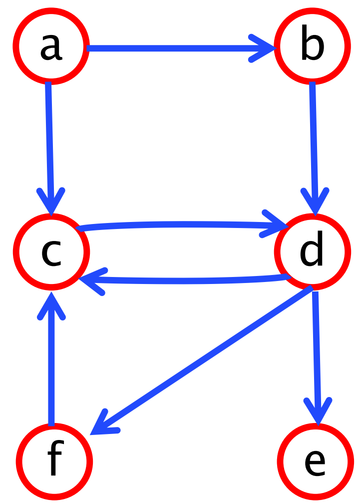
ordered: $e = (u,v)$ (*directed* graph / *digraph*)



Undirected

$V = \{A,B,C,D,E,F\}$

$E = \{\{A,B\},\{A,C\},\{B,C\},\{B,D\},\{C,D\},\{D,E\},\{D,F\},\{E,F\}\}$



Directed

$V = \{a,b,c,d,e,f\}$

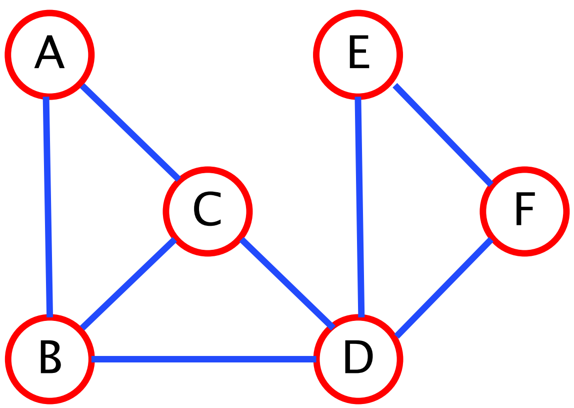
$E = \{(a,b),(a,c),(b,d),(c,d),(d,c),(d,e),(d,f),(f,c)\}$

Graph Terminology

- Size? Edges or vertices?
- Usually take size to be $n = |V|$ (# of vertices)
- But the runtime of graph algorithms often depend on the number of edges, $|E|$
- Relationships between $|V|$ and $|E|$?

Undirected Graphs: adjacency, degree

- If {u,v} is an edge, then u and v are *neighbors* (also: u is *adjacent* to v)
- *degree* of v = number of neighbors of v



Fact:

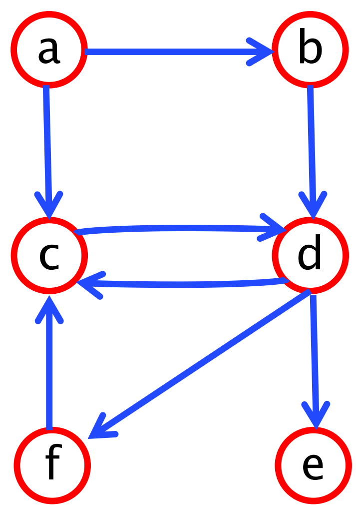
$$\sum_{v \in V} \deg(v) = 2|E|$$

(Why?)

Directed Graphs: adjacency, degree

- If (u,v) is an edge, then v is a *successor* of u and u is a *predecessor* of v

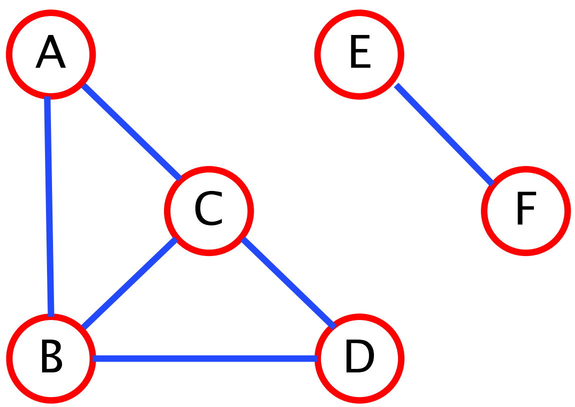
- *Out-degree* of v = number of successors of v
- *In-degree* of v = number of predecessors of v



Undirected Graphs: paths, connectivity

- A *path* is a list of unique vertices joined by edges.
  - For example, [a, c, d] is a path from a to d.

- A subgraph is *connected* if every pair of vertices in the subgraph has a path between them.



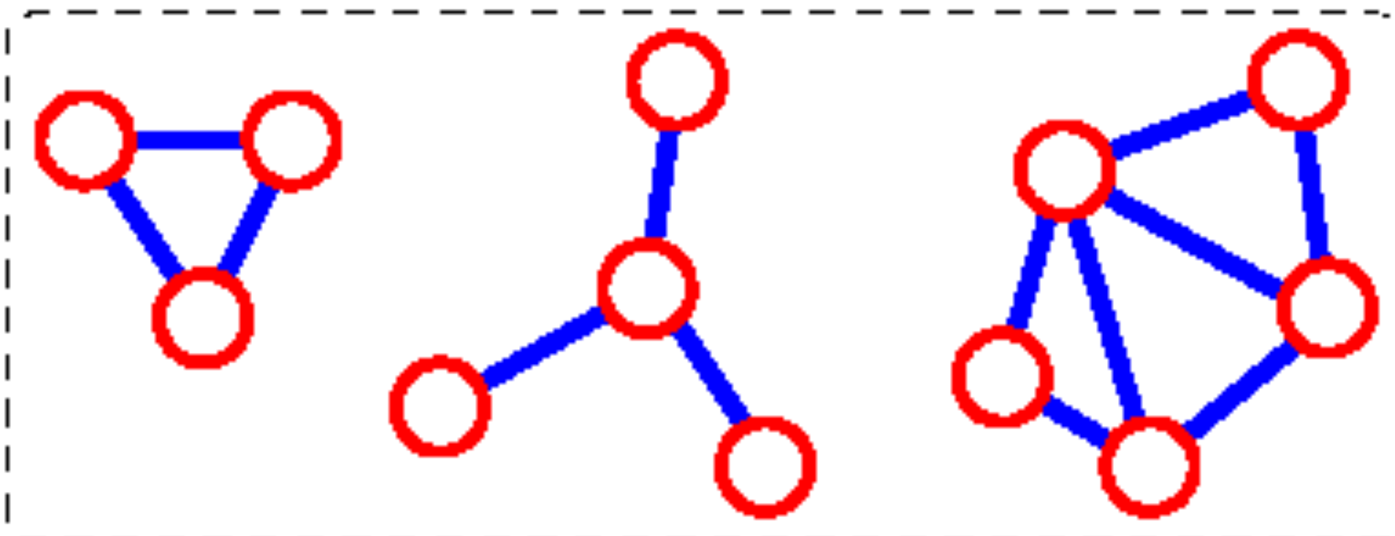
Not a connected graph.

Subgraph	Connected?
{A,B,C,D}	Yes
{E,F}	Yes
{C,D,E}	No
{A,B,C,D,E,F}	No

Undirected Graphs: components

(Connected) *component*: a maximal connected subgraph.

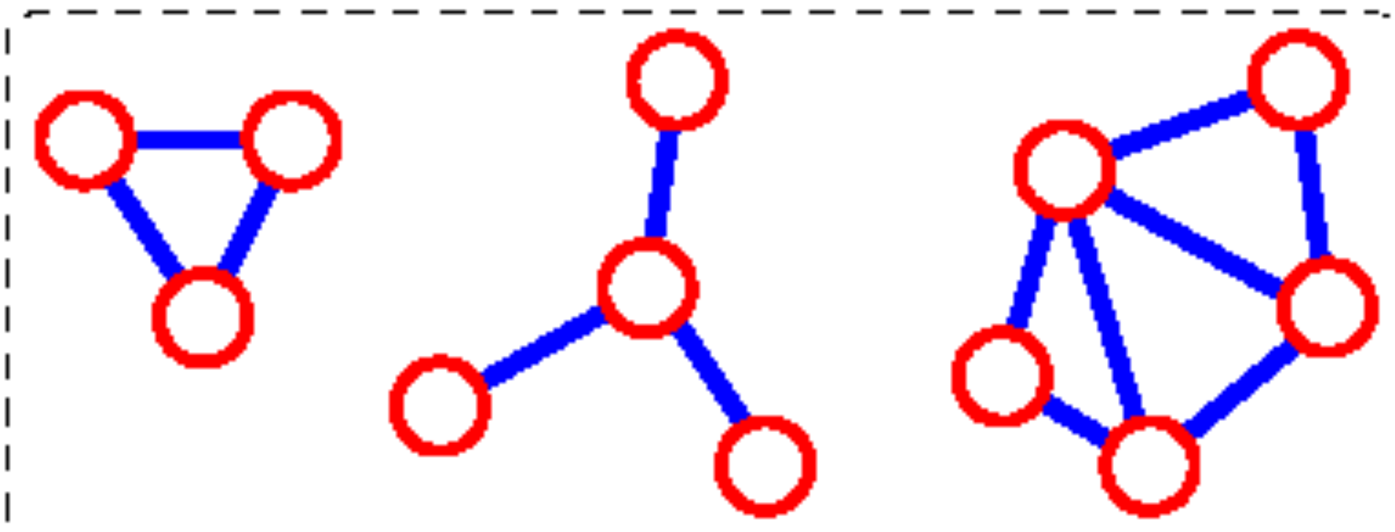
For example, this graph has 3 connected components:



Undirected Graphs: (mathematical) tree

Tree: connected acyclic graph (no cycles)

Example. Which component is a tree?



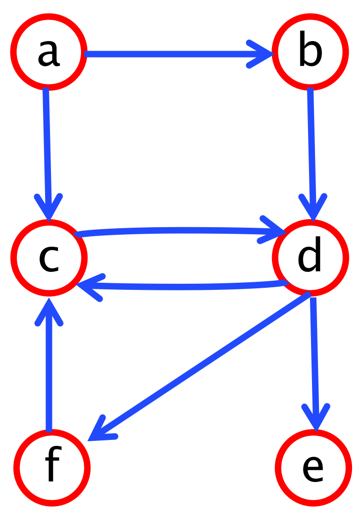
Question: for a tree, what is the relationship between m = #edges and n = #vertices?

$$m = n - 1$$

Directed Graphs: paths, connectivity

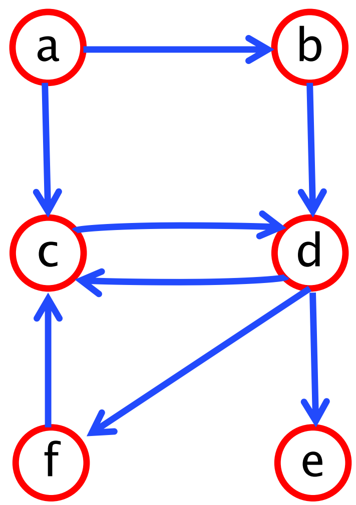
- A *directed path* is a list of unique vertices joined by directed edges.
  - For example, [a, c, d, f] is a directed path from a to f. We say f is *reachable* from a.

- A subgraph is *strongly connected* if for every pair (u,v) of its vertices, v is reachable from u and u is reachable from v.



Directed graphs: components

- *Strongly-connected component*: maximal strongly connected subgraph

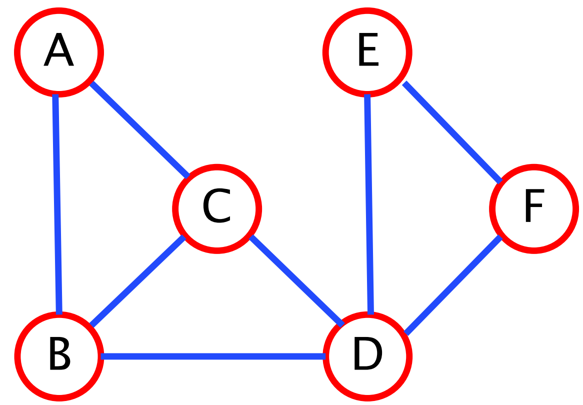


Strongly connected components
{a}
{b}
{c,d,f}
{e}

Viewing a graph as a data structure

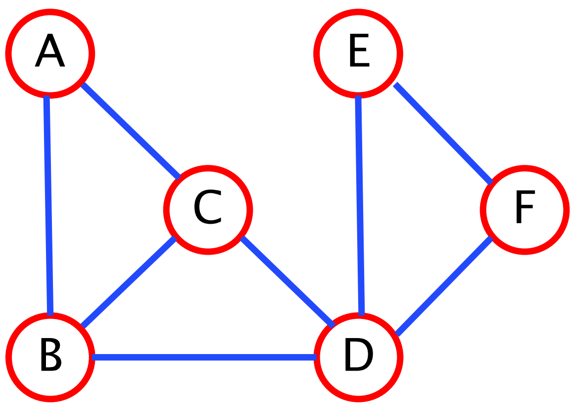
- Each vertex associated with a name (key)
- Examples:
  - City name
  - IP address
  - People in a social network
- An edge (undirected/directed) represents a link between keys
- Graphs are flexible: edges/nodes can have *weights*, *capacities*, or other attributes

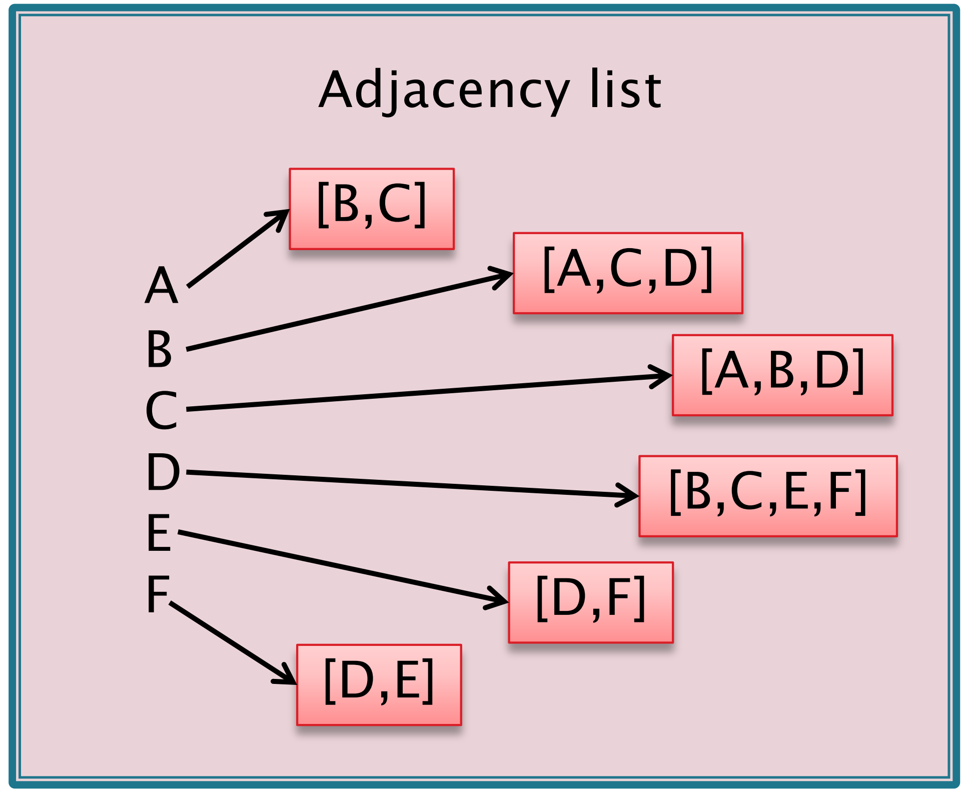
There are several alternatives for representing edges of a graph



- Edge list
  - A collection of vertices and a collection of edges
- Adjacency matrix
  - Each key is associated with an index from 0, …, (n-1)
    - Map from keys to ints?
  - Edges denoted by 2D array (#V x #V) of 0's and 1's
- Adjacency list
  - Collection of vertices
    - Map from keys to Vertex objects?
  - Each Vertex stores a List of adjacent vertices

Implementation tradeoffs





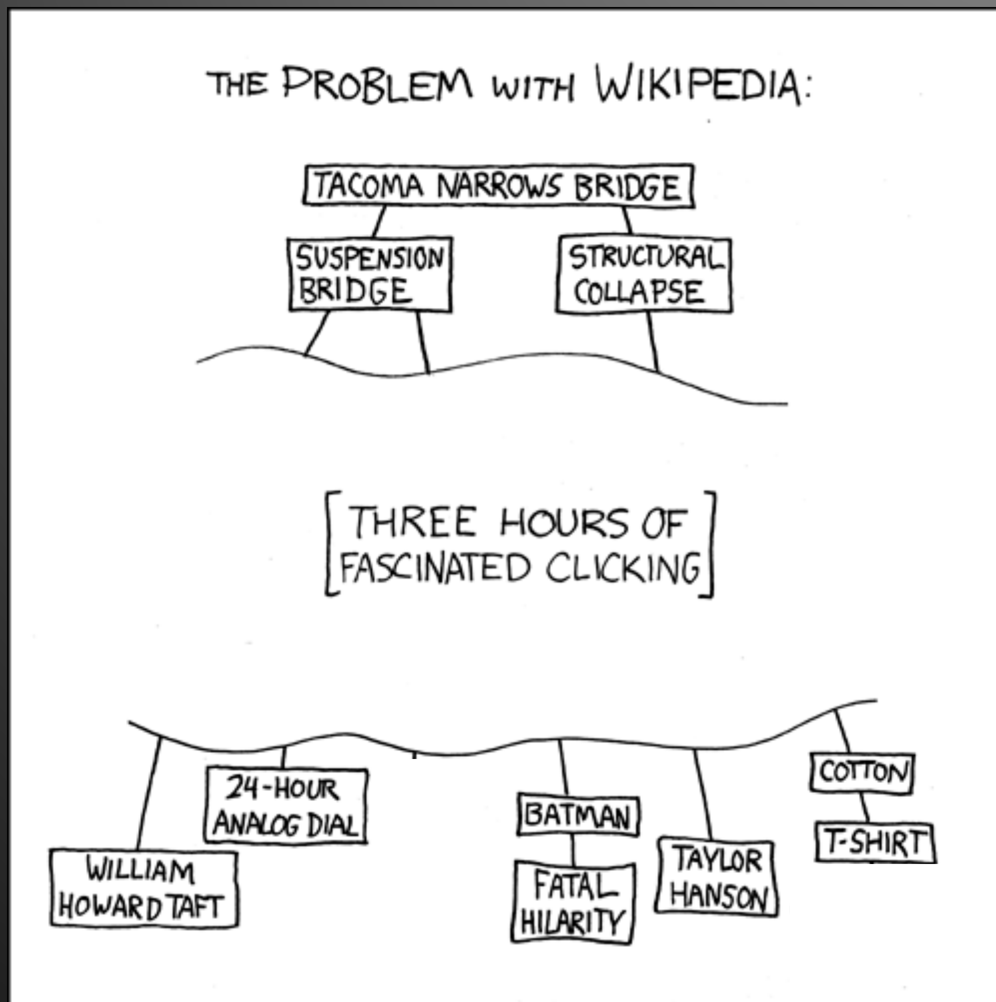
Adjacency matrix

A→0
B→1
C→2
D→3
E→4
F→5

	0	1	2	3	4	5
0	0	1	1	0	0	0
1	1	0	1	1	0	0
2	1	1	0	1	0	0
3	0	1	1	0	1	1
4	0	0	0	1	0	1
5	0	0	0	1	1	0

- Running time of degree(v)?
- Running time of deleteEdge(u,v)?
- Space efficiency?

GraphSurfing Project

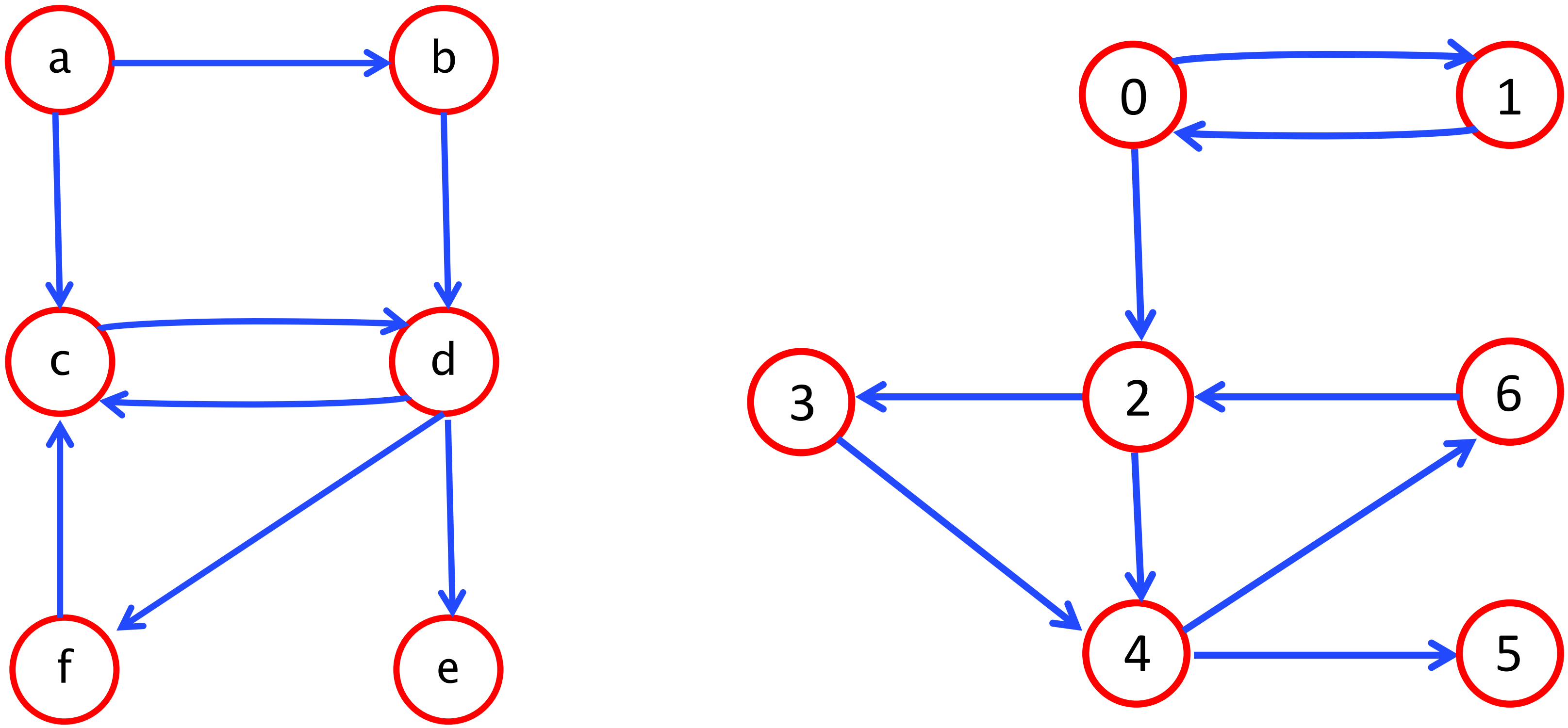


GraphSurfing assignment

- Milestone 1: Implement AdjacencyListGraph<T> and AdjacencyMatrixGraph<T>
  - both extend the given ADT, Graph<T>.
- Milestone 2: Write methods
  - stronglyConnectedComponent(v)
  - shortestPath(from, to)

    and use them to go WikiSurfing!

Example graphs for project

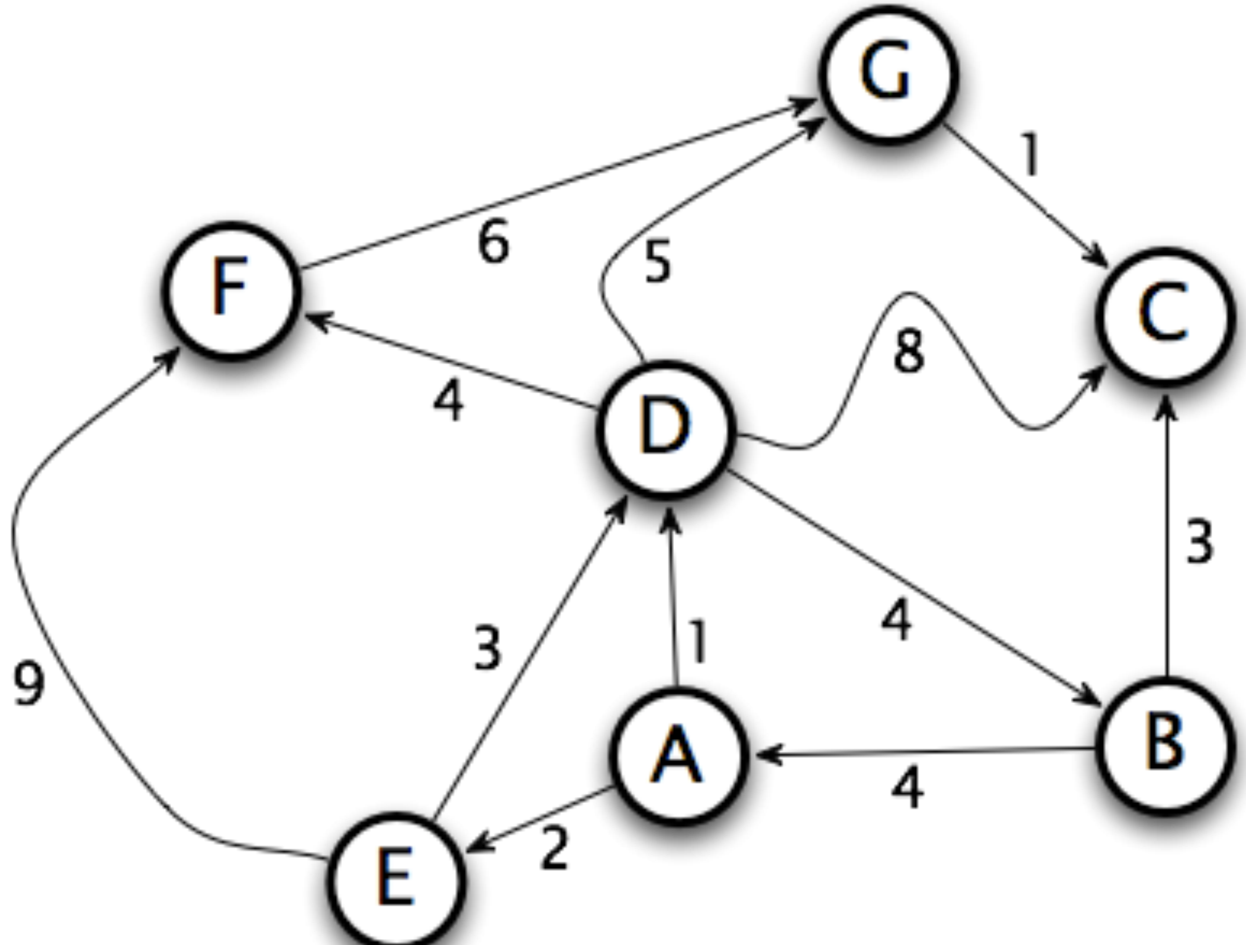


Sample Graph Problems

To discuss algorithms, take MA/CSSE473 or MA477

Weighted Shortest Path

- What's the cost of the shortest path from A to each of the other nodes in the graph?

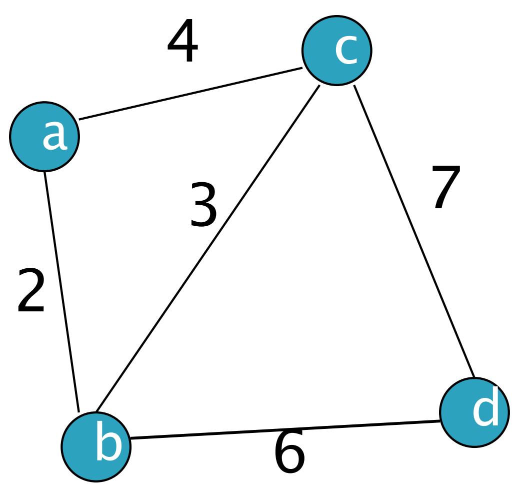


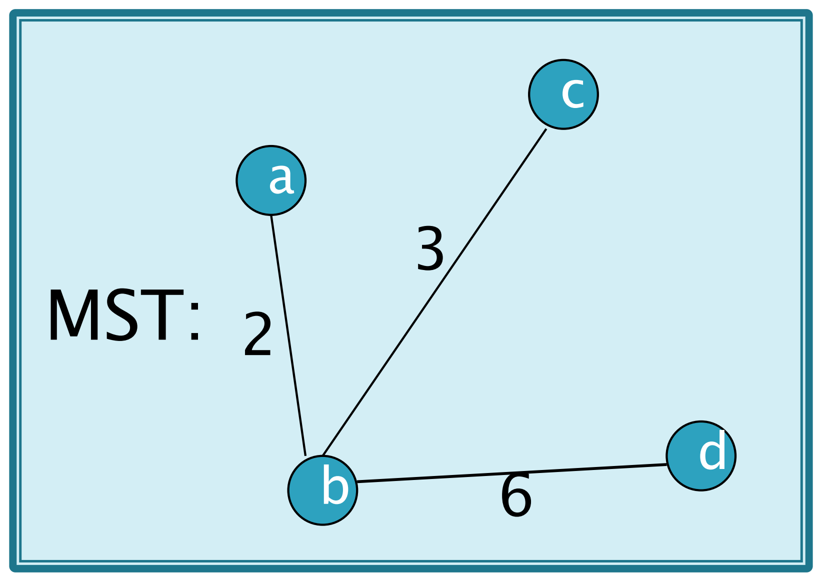
For much more on graphs, take MA/CSSE 473 or MA 477

Minimum Spanning Tree

- *Spanning tree*: a connected acyclic subgraph that includes all of the graph's vertices
- *Minimum spanning tree* of a weighted, connected graph: a spanning tree of minimum total weight

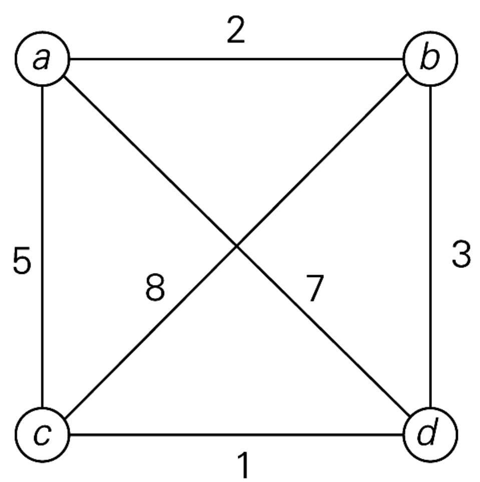
Example:





Traveling Salesman Problem (TSP)

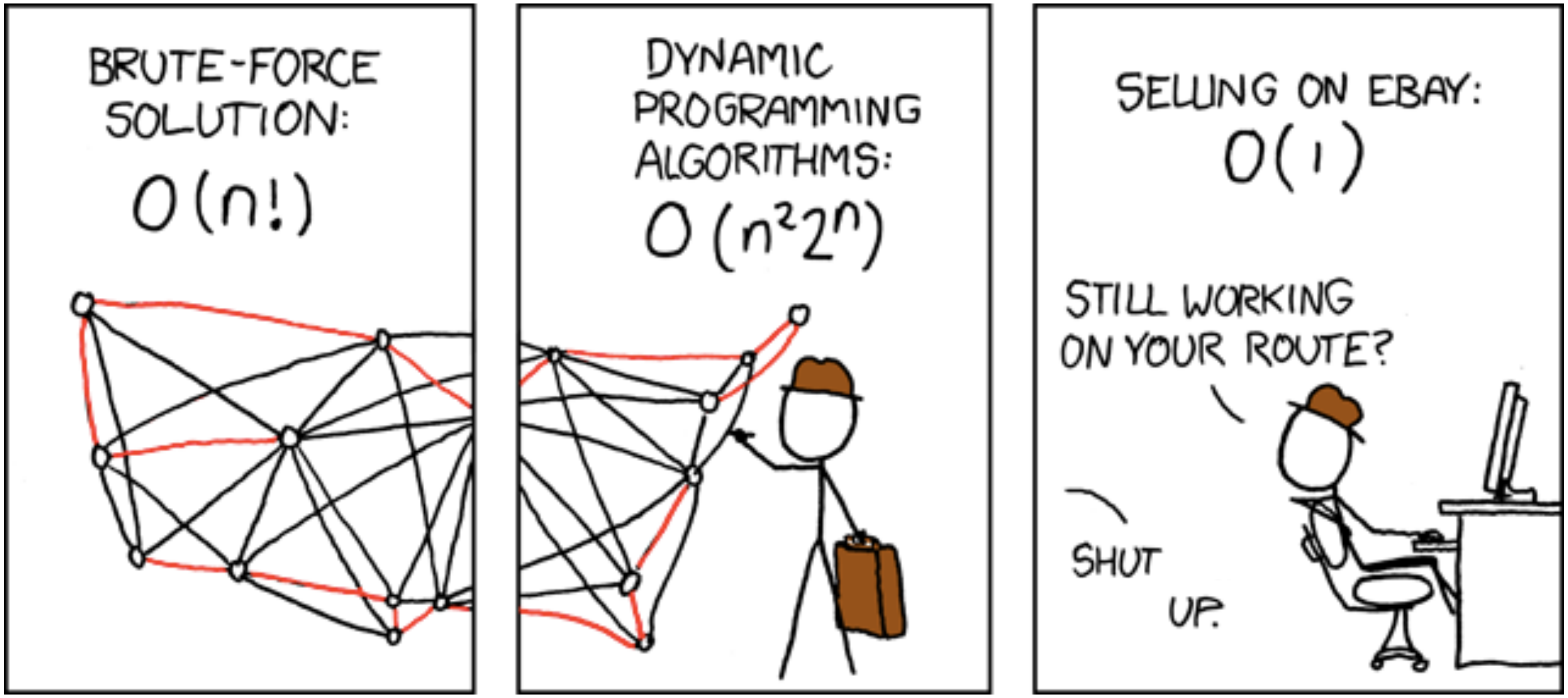
- *n* cities, weights are travel distance
- Must visit all cities (starting & ending at same place) with shortest possible distance



Tour	Length	
a —> *b* —> *c* —> *d* —> *a*	$l = 2 + 8 + 1 + 7 = 18$	
a —> *b* —> *d* —> *c* —> *a*	$l = 2 + 3 + 1 + 5 = 11$	optimal
a —> *c* —> *b* —> *d* —> *a*	$l = 5 + 8 + 3 + 7 = 23$	
a —> *c* —> *d* —> *b* —> *a*	$l = 5 + 1 + 3 + 2 = 11$	optimal
a —> *d* —> *b* —> *c* —> *a*	$l = 7 + 3 + 8 + 5 = 23$	
a —> *d* —> *c* —> *b* —> *a*	$l = 7 + 1 + 8 + 2 = 18$	

- Exhaustive search: how many routes?
- $(n-1)!/2 \in \Theta((n-1)!)$

Traveling Salesman Problem



- Online source for all things TSP:
  - http://www.math.uwaterloo.ca/tsp/