Q0–2

- Proof by Contradiction. Suppose there is such a MaxCSS, namely $S_{p,q}$, where $i+1 \le p \le j$.

i	$S_{i,j}$ just became negative!	j

Case 1. $q > j$

p	MaxCSS	q

Case 2. $q \le j$

p	MaxCSS	q

CSSE 230 Day 4

Maximum Contiguous Subsequence Sum

After today's class you will be able to:

provide an example where an insightful algorithm can be much more efficient than a naive one.

Announcements

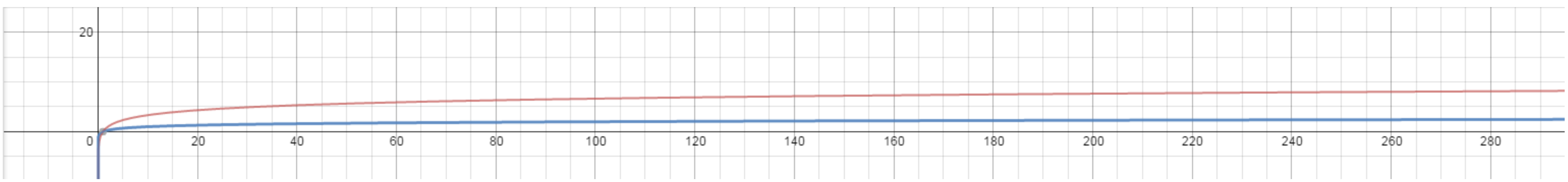
- ~~Sit with your StacksAndQueues partner now~~

- Why Math?

- HW2
  - Recommended: complete #1 (UML Diagram of Collections) by next class, as we will discuss Collections
  - You should be ready to work on all of the problems

Homework 2

- Is it true that $\log_a(n)$ is $\theta(\log_b(n))$?
- Complete homework 2 to find out the exciting conclusion!
- Here is the graph for a=2 and b=10:



- Is it true that 3^n is $\theta(2^n)$?

Andrew Hettlinger ▸ **Matt Boutell**
November 6 at 12:30pm ·

In your class, I never thought I'd actually use big O notation, but now I find myself using it in my complaints to coworkers about how a previous developer would sort a list before doing a binary search to find a single element O(nlogn) + O(logn) instead of just doing a linear search O(n). I feel really nerdy now (as if I didn't before 🙂)

Like · Comment

So why would we ever sort first to do binary search?

Recap: MCSS

- Problem definition: given a nonempty sequence of *n* (possibly negative) integers $A_0, A_1, A_2, \dots, A_{n-1}$, find the maximum contiguous subsequence

$$S_{i,j} = \sum_{k=i}^{j} A_k$$

and the corresponding values of i and j.

Q3

Recap: Eliminate the most obvious inefficiency, get $\Theta(N^2)$

```
for( int i = 0; i < a.length; i++ ) {
    int thisSum = 0;
    for( int j = i; j < a.length; j++ ) {
        thisSum += a[ j ];

        if( thisSum > maxSum ) {
            maxSum = thisSum;
            seqStart = i;
            seqEnd   = j;
        }
    }
}
```

- Exhaustive search: find every $S_{i,j}$

MCSS is $O(n^2)$

- Is MCSS $\theta(n^2)$?
  - Showing that a problem is Ω (g(n)) is much tougher. How do you prove that it is impossible to solve a problem more quickly than you already can?
  - Can we find a yet faster algorithm?
    - If so, it can't use exhaustive search. (Why?)

f(n) is O(g(n)) if $f(n) \leq cg(n)$ for all $n \geq n_0$
- So O gives an upper bound

f(n) is Ω(g(n)) if $f(n) \geq cg(n)$ for all $n \geq n_0$
- So Ω gives a lower bound

f(n) is θ(g(n)) if $c_1g(n) \leq f(n) \leq c_2g(n)$ for all $n \geq n_0$
- So θ gives a tight bound
- f(n) is θ(g(n)) if it is both O(g(n)) **and** Ω(g(n))

Q4

Observations?

- Consider {1, 4, −2, 3, −8, 4, −6, 5, −2}

- Any subsequences you can safely ignore?
  - Discuss with another student (1.5 minutes)

Observation 1

- We noted that a max–sum sequence $S_{i,j}$ cannot begin with a negative number.
- Generalizing this, it cannot begin with a prefix $S_{i,k}$ with $k<j$ whose sum is negative.
  - **Proof by contradiction.** Suppose that $S_{i,j}$ is a max–sum sequence and that $S_{i,k}$ is negative. In that case, a larger–sum contiguous sequence can be created by removing $S_{i,k}$. However, this violates our assumption that $S_{i,j}$ is a max–sum contiguous sequence.

Q6

Observation 2

- Every contiguous subsequence that borders a maximum contiguous subsequence must have a negative or zero sum.
  - **Proof by contradiction.** Consider a contiguous subsequence that borders an MCSS sequence. Suppose it has a positive sum. We can then create a larger max-sum sequence by combining the two adjacent sequences. This contradicts our assumption that the original sequence has the maximum sum.

Q7

Observation 3

- Imagine we are growing subsequences from a fixed left index *i*. That is, we compute the sums $S_{i,j}$ for increasing *j*.

- Claim: If there is such an $S_{i,j}$ that "just became negative" (for the first time, with the inclusion of the j^{th} term), any subsequence starting in between $i + 1$ and j cannot be a MaxCSS (unless its sum equals an already-found MaxCSS)!

- In other words, as soon as we find that $S_{i,j}$ is negative, we can skip all sums that begin with any of $A_{i+1}, \ldots, A_j$.

- I.e., we can "skip i ahead" to be $j + 1$.

Proof of Observation 3

- Proof by Contradiction. Suppose there is such a MaxCSS, namely $S_{p,q}$, where $i+1 \le p \le j$.

i $S_{i,j}$ just became negative! j

- Key point. What must be true of the following sums?

$S_{i,p-1} \ge 0$ | $S_{p,j} < 0$

$S_{i,j}$ just became negative!

Case 1. $q > j$

p MaxCSS q

Starts with a negative prefix. Violates Obs. 1!

Case 2. $q \le j$

p MaxCSS q

Borders a subsequence with nonnegative sum.
Violates Obs. 2, or there is a previous MaxCSS with the same sum.

Q8

New, improved code!

```
public static Result mcssLinear(int[] seq) {
    Result result = new Result();
    result.sum = 0;
    int thisSum = 0;

    int i = 0;
    for (int j = 0; j < seq.length; j++) {
        thisSum += seq[j];

        if (thisSum > result.sum) {
            result.sum = thisSum;
            result.startIndex = i;
            result.endIndex = j;
        } else if (thisSum < 0) {
            // advances start to where end
            // will be on NEXT iteration
            i = j + 1;
            thisSum = 0;
        }
    }
    return result;
}
```

$S_{i,j}$ is negative. So, skip ahead per Observation 3

Running time is O (?)
How do we know?

What have we shown?

- MCSS is O(n)!
- Is MCSS Ω(n) and thus θ(n)?
  - Yes, intuitively: we must at least examine all n elements

Time Trials!

- From personal repo, checkout `MCSSRaces`
- Study code in `MCSS.main()`
- For each algorithm, how large a sequence can you process on your machine in less than 1 second?

MCSS Conclusions

- The first algorithm we think of may be a lot worse than the best one for a problem
- Sometimes we need clever ideas to improve it
- Showing that the faster code is correct can require some serious thinking
- Programming is more about careful consideration than fast typing!

Interlude

- If GM had kept up with technology like the computer industry has, we would all be driving $25 cars that got 1000 miles to the gallon.
  – Bill Gates

- If the automobile had followed the same development cycle as the computer, a Rolls-Royce would today cost $100, get a million miles per gallon, and explode once a year, killing everyone inside.
  – Robert X. Cringely

Interlude

Stacks and Queues

A preview of Abstract Data Types and Java Collections

This week's major program

Q10

Stacks and Queues assignment

Intro: Ideas for how to implement stacks and queues using arrays and linked lists

How to write your own growable circular queue:

1. Grow it as needed (like day 1exercise)
2. Wrap-around the array indices for more efficient dequeuing

Q11

Stacks and Queues implementation

Analyze implementation choices for Queues – much more interesting than stacks! (See HW)

Application: An exercise in writing cool algorithms that evaluate mathematical expressions:

Evaluate Postfix: 6 7 8 * +
(62. How?)
Convert Infix to Postfix: 6 + 7 * 8
(6 7 8 * + You'll figure out how)

Both using **stacks.**
Read assignment for hints on *how*.