

CSSE230 – MCSS $O(N^3)$ Version

Example:

$a = [-2, 11, -4, 13]$

$|a| = 4$, so $N = 4$

[] i j k	# of iterations of innermost loop	Summation for # of times inner loop executed
0 0 0 0 1 0 0 1 1 0 2 0 0 2 1 0 2 2 0 3 0 0 3 1 0 3 2 0 3 3	1 2 3 4	$\sum_l^N l = \frac{N(N+1)}{2}$ $N = 4, \text{ so } \frac{4(5)}{2} = 10$
1 1 1 1 2 1 1 2 2 1 3 1 1 3 2 1 3 3	1 2 3	$\sum_l^{N-1} l = \frac{(N-1)(N)}{2}$ $N = 4, \text{ so } \frac{3(4)}{2} = 6$
2 2 2 2 3 2 2 3 3	1 2	$\sum_l^{N-2} l = \frac{(N-2)(N-1)}{2}$ $N = 4, \text{ so } \frac{2(3)}{2} = 3$
3 3 3	1	$\sum_l^{N-3} l = \frac{(N-3)(N-2)}{2}$ $N = 4, \text{ so } \frac{1(2)}{2} = 1$
Closed Forms Required $\sum_{k=0}^n k = \frac{n(n+1)}{2}$ $\sum_{k=0}^n k^2 = \frac{n(n+1)(2n+1)}{6}$		$\sum_{m=1}^N \frac{m(m+1)}{2}$ <p>For $N = 4$, inner loop executed: $\sum_{m=1}^4 \frac{m(m+1)}{2} = 1 + 3 + 6 + 10 = 20$</p> <p>In General: $\sum_{m=1}^N \frac{m(m+1)}{2} = \sum_{m=1}^N \frac{m^2 + m}{2} = \frac{1}{2} \sum_{m=1}^N m^2 + \frac{1}{2} \sum_{m=1}^N m = \frac{N(N+1)(N+2)}{6}$</p>