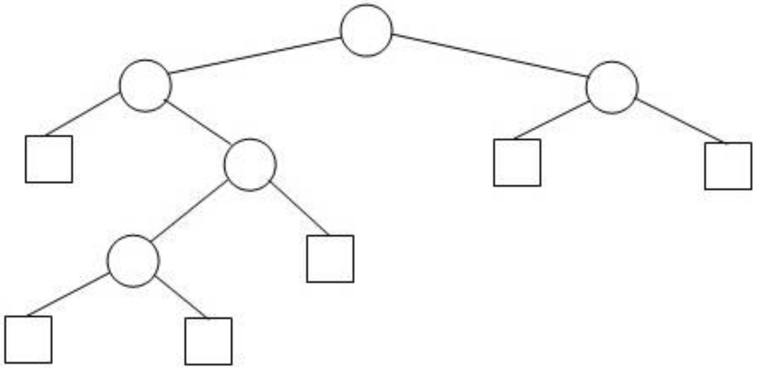
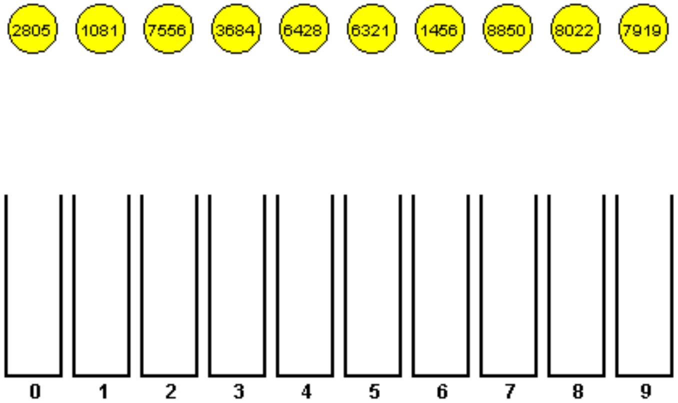
CSSE 230

Sorting Lower Bound
Radix Sort



What is the min height of a tree with X external nodes?

Radix sort to the rescue … sort of…



After today, you should be able to…
…explain why comparison-based sorts need at least O(n log n) time
… explain bucket sort
… explain radix sort
… explain the situations in which radix sort is faster than O(n log n)

http://www.cs.auckland.ac.nz/software/AlgAnim/radixsort.html

Announcements

- SortingRaces is due Friday.
  Day 29’s class time will be SortingRaces work time.

- The sounds of sorting. Radix sort later.
  - https://www.youtube.com/watch?v=kPRA0W1kECg

A Lower-Bound on Sorting Time

We can’t do much better than what we already know how to do.

What’s the best best case?

- Lower bound for best case?

- A particular algorithm that achieves this?

What’s the best worst case?

- Want a function f(N)
  such that the **worst case running time**
  for all sorting algorithms is Ω(f(N))

- How do we get a handle on
  “all sorting algorithms”?

Tricky!

What are “all sorting algorithms”?

- We can’t list all sorting algorithms and analyze all of them
  - Why not?

- But we can find a **uniform representation** of any sorting algorithm that is based on **comparing** elements of the array to each other

First of all...

- The problem of sorting N elements is at least as hard as determining their ordering
  - e.g., determining that $a_3 < a_4 < a_1 < a_0 < a_2$

0	1	2	3	4
58	55	73	5	10

  - sorting = determining order, then movement

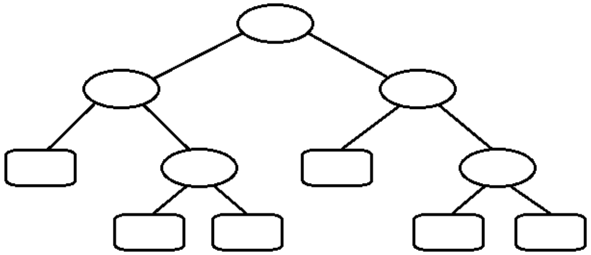
- So any lower bound on all "order-determination" algorithms is also a lower bound on "all sorting algorithms"

Q1

Sort Decision Trees

- Let A be any **comparison-based algorithm** for sorting an array of distinct elements

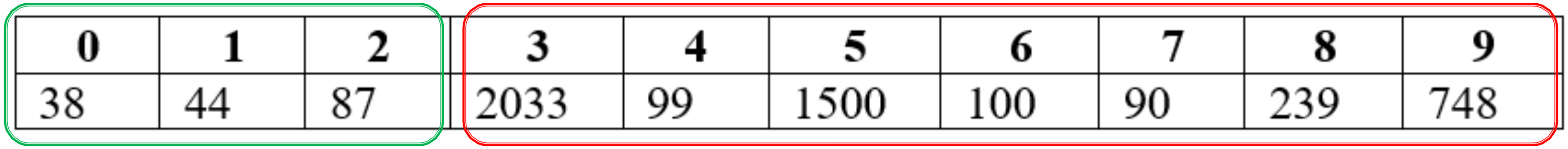
- We can draw an EBT that corresponds to the comparisons that will be used by A to sort an array of N elements
  - This is called a **sort decision tree**
  - Internal nodes are comparisons
  - External nodes are orderings



  - Different algorithms will have different trees

Insertion Sort

- Basic idea:
  - Think of the array as having a sorted part (at the beginning) and an unsorted part (the rest)



0	1	2	3	4	5	6	7	8	9
38	44	87	2033	99	1500	100	90	239	748

  - Get the first value in the unsorted part
  - Insert it into the correct location in the sorted part, moving larger values up to make room

Repeat until unsorted part is empty

Q2–4

So what?

- Minimum number of external nodes in a sort decision tree? (As a function of N)
- Is this number dependent on the algorithm?
- What’s the height of the shortest EBT with that many external nodes?

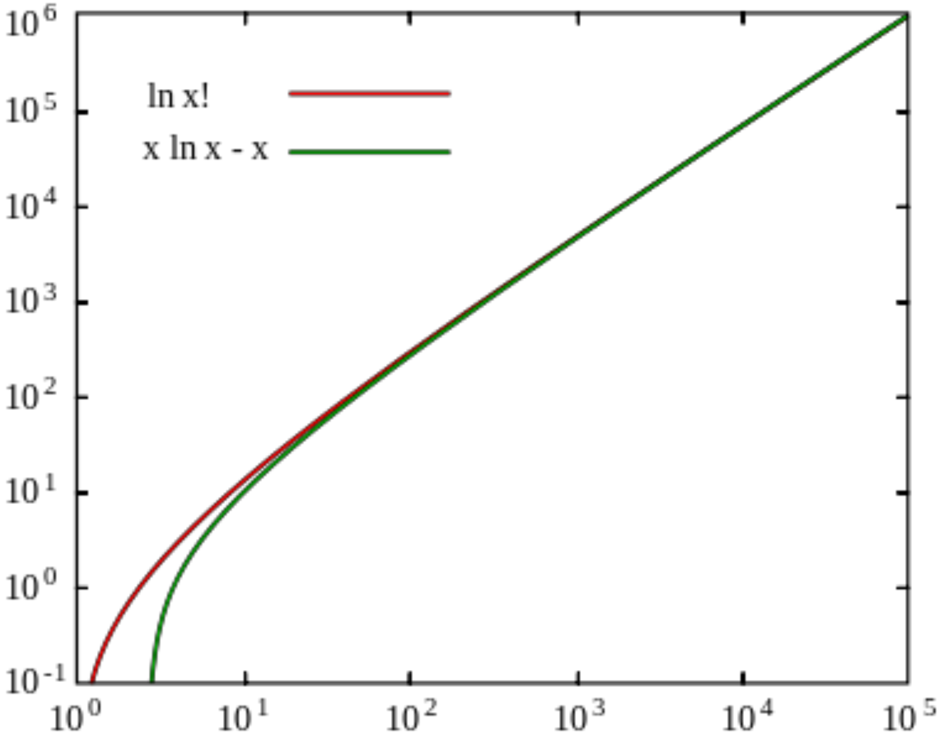
$$\lceil \log N! \rceil \approx N \log N - 1.44N = \Omega(N \log N)$$

No comparison-based sorting algorithm, known or not yet discovered, can **ever** do better than this!

An approximation for log (n!)

- Use **Stirling's approximation**:

$$\ln n! = n \ln n - n + O(\ln(n))$$



http://en.wikipedia.org/wiki/Stirling%27s_approximation

Can we do better than *N* log *N*?

- Ω(N log N) is the best we can do if we compare items

- Can we sort without comparing items?

Yes, we can! We can avoid comparing items and still sort. This is fast if the range of data is small.

Q5

- Observation:
  - For N items, if the range of data is less than N, then we have duplicates
- O(N) sort: Bucket sort
  - Efficient if possible values come from limited range and have a uniform distribution over the range
  - Example: Exam grades histogram
- A variation: Radix sort

Q6-7

Radix sort

- A picture is worth 10^3 words, but an animation is worth 2^{10} pictures, so we will look at one.
- http://www.cs.auckland.ac.nz/software/AlgAnim/radixsort.html (good but blocked)

- https://www.youtube.com/watch?v=xuU-DS_5Z4g&src_vid=4S1L-pyQm7Y&feature=iv&annotation_id=annotation_133993417 (video, good basic idea, distracting zooms)

- http://www.cs.usfca.edu/~galles/visualization/RadixSort.html (good, uses single array)

Q8–10

RadixSort is almost O(n)

- It is O(kn)
  - Looking back at the radix sort algorithm, what is k?

- Look at some extreme cases:
  - If all integers in range 0–99 (so, many duplicates if N is large), then k = _____

  - If all N integers are distinct, k = ____

Radix sort example: card sorter

Used an appropriate combo of mechanical, digital, and human effort to get the job done.

Type 82 Electric Punched Card Sorting Machine

http://en.wikipedia.org/wiki/IBM_card_sorter